The Bridge to A level

PiXL Y11 Maths Bridging Unit

This pack contains a programme of activities and resources to prepare you to start A-level in Maths in September.

It is aimed to be used after you complete your GCSE throughout the remainder of the summer term and over the summer holidays to ensure you are ready to start your course in September.

The resources include:

1. Links to websites where you can research the topics you will be exploring in your sixth form courses and get a flavour of mathematics beyond GCSE.
2. 15 key pre-knowledge topics that will help you to be successful in your course. The topics covered are a mixture of GCSE topics, and topics which extend GCSE but which will be very useful on your A level course.
3. A diagnostic assessment that will test your key knowledge of these 15 topics.
4. Suggested therapies to help you with those topics with which you are having difficulty.
5. A second assessment which will test the effectiveness of your use of the therapies.
6. A bank of problem solving questions to accompany each of the 15 topics to test that you can apply the skills that you have learned.

Websites

NRich

http://nrich.maths.org/secondary-upper

Mathwire

http://mathwire.com/archives/enrichment.html

The History of Maths – Wikipedia

https://en.wikipedia.org/wiki/History_of_mathematics

The History of Maths – Youtube video

https://www.youtube.com/watch?v=cy-8lPVKLIo

15 key Topics

1	Solving quadratic equations
2	Changing the subject
3	Simultaneous equations
4	Surds
5	Indices
6	Properties of Lines
7	Sketching curves
8	Transformation of functions
9	Pythagoras' theorem and Trigonometric ratios
10	Sine / Cosine Rule
11	Inequalities
12	Proof
13	Vectors
14	Probability
15	Statistics

The Bridge to A level

Diagnosis

1 Solving quadratic equations

Question 1

Solve $x^2 + 6x + 8 = 0$

(2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

(1)

Total / 10

2 Changing the subject

Question 1

Make v the subject of the formula E = $\frac{1}{2}$ mv^2

(3)

Question 2

Make r the subject of the formula V = $\frac{4}{3}\Pi$ r^2

(3)

Question 3

Make c the subject of the formula P = $\frac{C}{C+4}$

(4)

Total / 10

3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$

(3)

Question 2

Find the coordinates of the point of intersection of the lines $5x + 2y = 20$ and $y = 5 - x$

(3)

Question 3

Solve the simultaneous equations

$$x^2 + y^2 = 5$$

$$y = 3x + 1$$

(4)

Total / 10

4 Surds

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

(3)

Total / 10

5 Indices

Question 1

Simplify the following

(i) a^0

(1)

(ii) $a^6 \div a^{-2}$

(1)

(iii) $(9a^6b^2)^{-0.5}$

(3)

Question 2

(i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$

(2)

(ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$

(3)

Total / 10

6 Properties of Lines

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

(3)

(ii) Find the length of AC

(2)

Question 2

Find, in the form $y = mx + c$, the equation of the line passing through A (3,7) and B (5,-1).

Show that the midpoint of AB lies on the line $x + 2y = 10$

(5)

Total / 10

7 Sketching curves

Question 1

In the cubic polynomial f(x), the coefficient of x^3 is 1. The roots of f(x) = 0 are -1, 2 and 5.

Sketch the graph of y = f(x)

(3)

Question 2

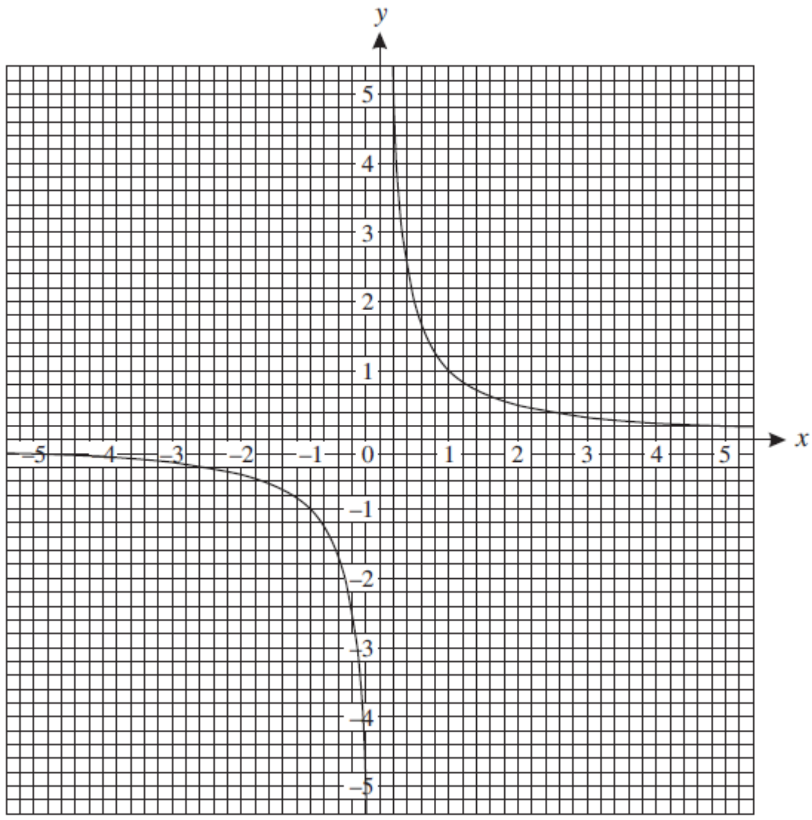
Sketch the graph of $y = 9 - x^2$

(3)

Question 3

The graph below shows the graph of $y = \frac{1}{x}$

On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \leq x \leq 5$



(4)

Total / 10

8 Transformation of functions

Question 1

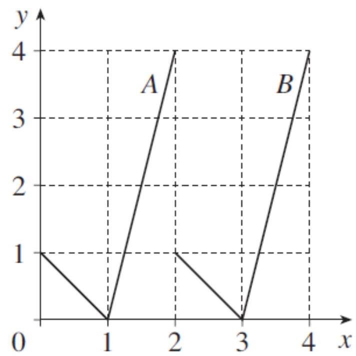
The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Write down an equation for the translated curve. You need not simplify your answer.

(2)

Question 2

This diagram shows graphs A and B.



(i) State the transformation which maps graph A onto graph B

(2)

(ii) The equation of graph A is $y = f(x)$.

Which one of the following is the equation of graph B ?

$y = f(x) + 2$	$y = f(x) - 2$	$y = f(x+2)$	$y = f(x-2)$
$y = 2f(x)$	$y = f(x+3)$	$y = f(x-3)$	$y = 3f(x)$

(2)

Question 3

(i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

(2)

(ii) Sketch the graph of $y = x^2 - 4$

(2)

Total / 10

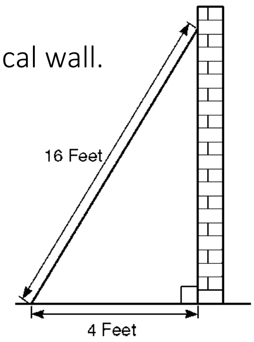
9 Trigonometric ratios

Question 1

Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall.

The ladder is 16 feet long.

The foot of the ladder is 4 feet from the base of the wall.



(i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

(2)

(ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 significant figures

(2)

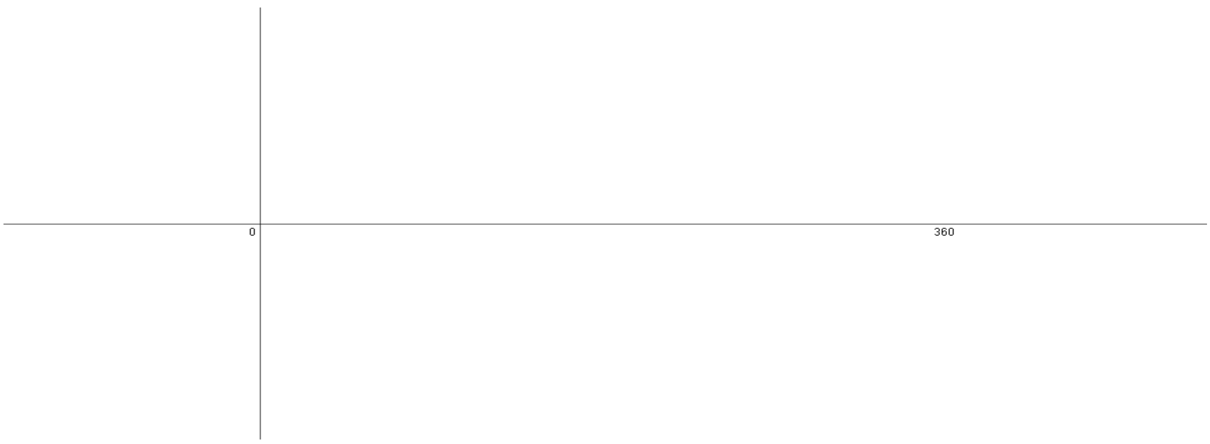
Question 2

Given that cos Θ = $\frac{1}{3}$ and Θ is acute, find the exact value of tan Θ

(3)

Question 3

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360°$

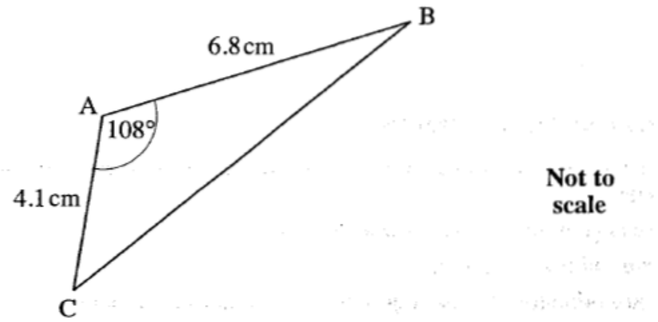


(3)

Total / 10

10 Sine / Cosine Rule

Question 1



For triangle ABC, calculate

(i) the length of BC

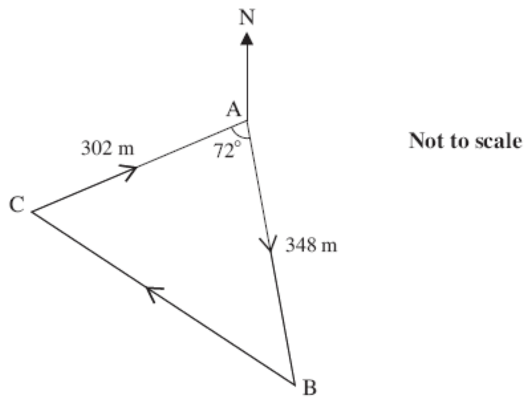
(3)

(ii) the area of triangle ABC

(3)

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Calculate the total length of the course for this race.

(4)

Total / 10

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11 Inequalities

Question 1

Solve

a) $x^2 - 36 \leq 0$

b) $9x^2 - 25 \geq 0$

c) $3x^2 + 10x < 0$

(3)

Question 2

Solve $\frac{21}{x+2} - \frac{5}{x+1} < 4$

....

(4)

Question 3

Solve $3x^2 - 8 > 2x$

....

(3)

Total / 10

12 **Algebraic proof**

Question 1

a) If n is a positive integer, write down expressions for the next two consecutive integers.

(1)

b) Use algebra to prove that the sum of three positive consecutive integers is always a multiple of 3.

(3)

Question 2

Prove that the square of an odd number is also odd.

(3)

Question 3

Given that x is a positive integer, prove that $\frac{4x^3+20x}{2x^2+10}$ is always even.

(3)

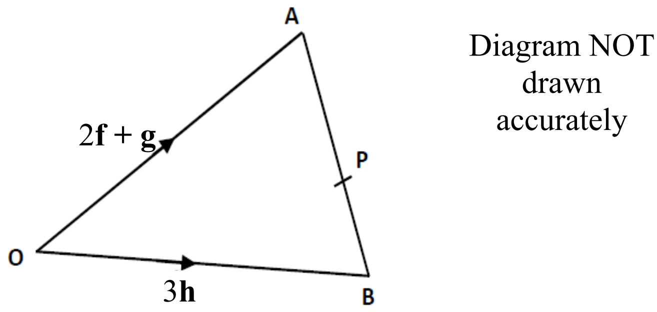
Total / 10

13 <u>Vectors</u>

Question 1

OAP is a triangle

$\overrightarrow{OA} = 2\mathbf{f} + \mathbf{g}$ and $\overrightarrow{OB} = 3\mathbf{h}$
P is the point on AB such that AP: PB = 2:1



(a) Find the vector $\overrightarrow{BA}$ in terms of **f**, **g** and **h**.

....

(1)

(b) Find the vector $\overrightarrow{PO}$ in terms of **f**, **g** and **h**

....

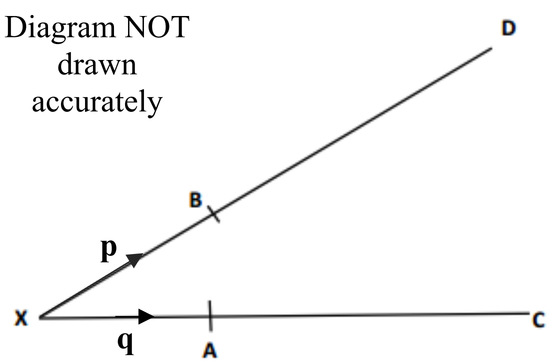
(2)

Question 2

B is the point on AD such that XB:BD is 1:2

A is the point on XC such that XA:XC is 1:2

$\overrightarrow{XB} = \mathbf{p}$ and $\overrightarrow{XA} = \mathbf{q}$



Use vectors to explain the geometrical relationships between the line segments BA and DC.

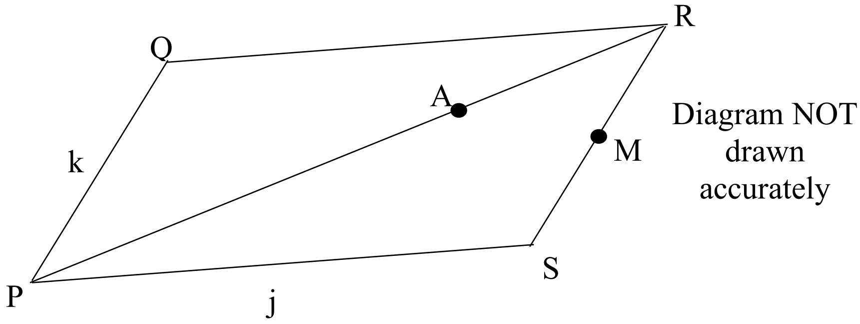
(4)

Question 3

PQRS is a parallelogram.

A is the point on PR such that PA:AR is 2:1

M is the midpoint of RS.



(b) Prove that Q, A and M are co-linear.

....

(3)

Total / 10

14 Probability

Question 1

A box contains 3 new batteries, 5 partly used batteries and 4 dead batteries.

Kelly takes two batteries at random.

Work out the probability that she picks two different types of batteries.

....

(3)

Question 2

Caleb either walks to school or travels by bus.

The probability that he walks to school is 0.75.

If he walks to school, the probability that he will be late is 0.3.

If he travels to school by bus, the probability that he will be late is 0.1.

Work out the probability that he will not be late.

....

(3)

Question 3

The two way table shows the number of deaths and serious injuries caused by road traffic accidents in Great Britain in 2013.

		Speed Limit			
		20 mph	**30 mph**	**40 mph**	**Total**
Type of Injury	**Fatal**	6	520	155	681
	Serious	420	11582	1662	13664
	Total	426	12102	1817	14345

Work out an estimate for the probability:

(a) that the accident is serious.

....

(1)

(b) that the accident is fatal given that the speed limit is 30 mph.

....

(1)

(c) that the accident happens at 20 mph given that the accident is serious.

....

(2)

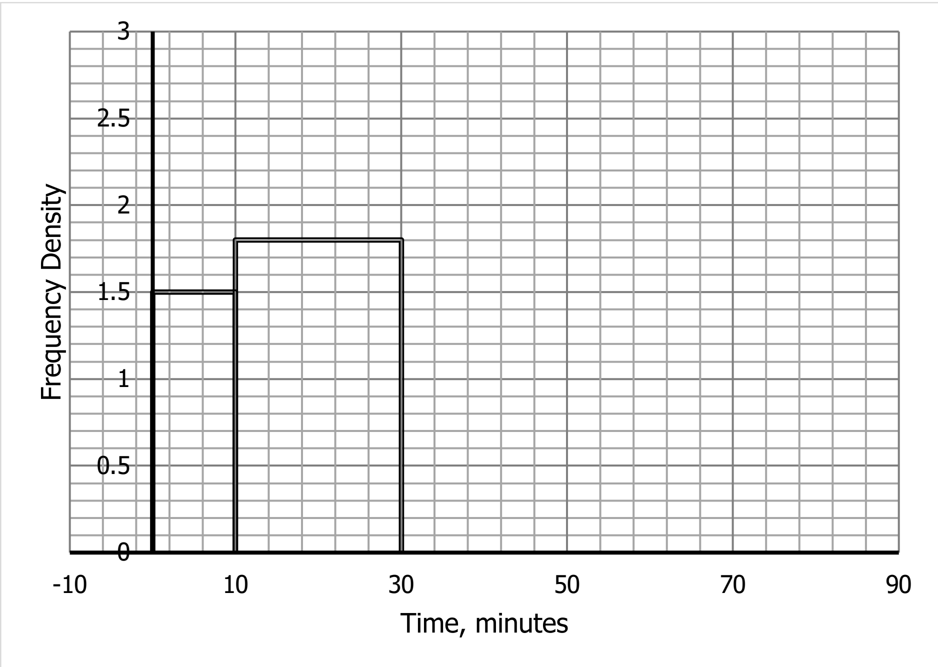
Total / 10

15 Statistics

Question 1

The histogram and the frequency table show some information about how much time vehicles spent in a car park.

Time, minutes	Frequency
$0 < x \le 10$	
$10 < x \le 30$	
$30 < x \le 60$	75
$60 < x \le 80$	24
Total	150



a) Use the information to complete the histogram

(2)

b) Use the histogram to find the missing frequencies in the table

....

(2)

Question 2

The table shows the length of 678 phone calls made at a call centre

Time, secs	Frequency
$0 < x \leq 20$	20
$20 < x \leq 60$	148
$60 < x \leq 120$	240
$120 < x \leq 300$	270
Total	678

a) Draw a fully labelled histogram to show the length of the phone calls.

(4)

b) Estimate the number of phone calls that lasted more than 4 minutes.

....

(2)

Total / 10

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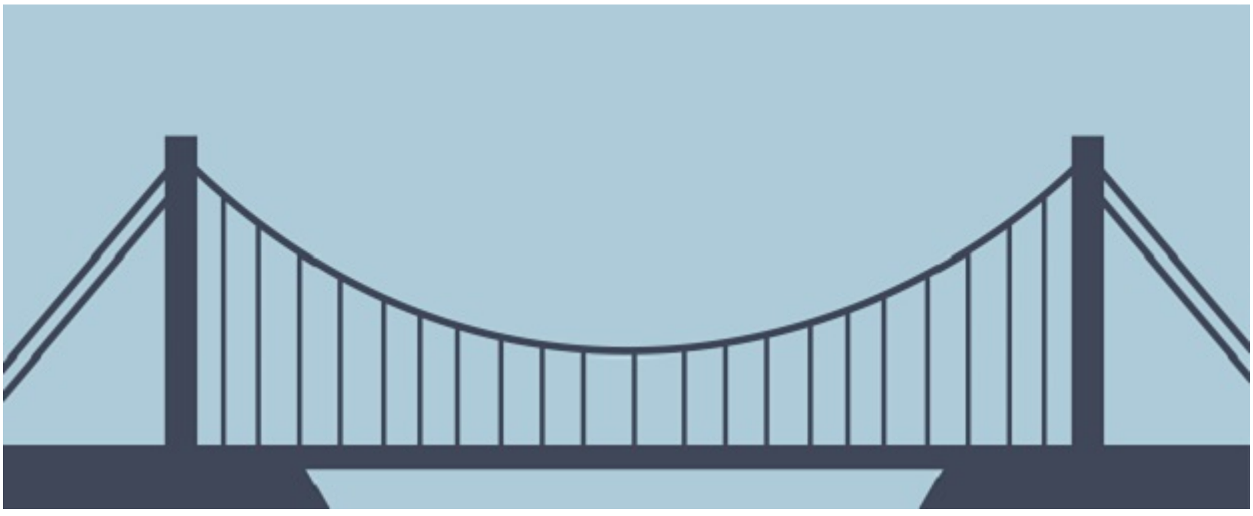
Diagnosis Mark Scheme



Section	Question	Answer	Marks	Notes
1	1	-2, -4	M1 A1	(x ± 2)(x ± 4)
	2	y = 3 or y = 4 cao x = $\pm\sqrt{3}$ or x = ± 2 cao	M1 A1 B2	For (y-3)(y+4) oe eg use of quad form y = 3 or y = 4 cao B1 for two roots correct or ft 'their' y B2 for cao
	3(i)	$(x-3)^2 - 7$	B1 M1A1	$(x-3)^2$ -7
	3(ii)	(3,-7)	B1	ft from part (i)
2	1	v = $\sqrt{\frac{2E}{m}}$ cao www	B3	Award M1 for a correct first constructive step, M2 for $v^2 = \frac{2E}{m}$ oe
	2	r = $\sqrt[3]{\frac{3V}{4\Pi}}$	B3	Award M2 for $r^3 = \frac{3V}{4\Pi}$, M1 for cube root of 'their' r^3
	3	C = $\frac{4P}{1-P}$ oe	M1 M1 M1 A1	PC + 4P = C 4P = C – PC 4P = C(1 – P)
3	1	(0.3,1.9)	M1 A1A1	for substitution or for rearrangement one mark each coordinate
	2	$(\frac{10}{3}, \frac{5}{3})$	M1 A1A1	for substitution or for rearrangement one mark each coordinate Note: award B2 if roiunded to 1dp or worse
	3	$(\frac{2}{5}, \frac{11}{5})$ or (-1,-2) or answer given as x=, y=	M1 M1 A1A1	substituting linear into non-linear forming quadratic in x one mark for each set of solutions
4	1(i)	7	M1 A1	9-2
	1(ii)	$\frac{5}{7} + \frac{4}{7}\sqrt{2}$	M1 M1 A1	multiplying top and bottom by $3 + \sqrt{2}$ $\frac{3+2+3\sqrt{2}+\sqrt{2}}{7}$ if one (or none) error only
	2(i)	$30\sqrt{2}$	M1 A1	for $\sqrt{8} = 2\sqrt{2}$ or $\sqrt{50} = 5\sqrt{2}$
	2(ii)	$\frac{1}{11} + \frac{2}{11}\sqrt{3}$	M1 M1 A1	multiplying top and bottom by $6 + \sqrt{3}$ denominator = 11 (or 33)

5	1(i)	1	B1	
	1(ii)	a^8	B1	
	1(iii)	$\dfrac{1}{3a^3b}$	B1 B1 B1	3b a^3 inverse
	2(i)	±5	M1 A1	for $\sqrt{25}$ or $\frac{1}{5}$ seen
	2(ii)	$8x^{10}y^{13}z^4$ (or $2^3x^{10}y^{13}z^4$)	B3	B2 for 3 elements correct B1 for 2 elements correct
6	1(i)	Grad AB = 1 Grad BC = -1 product of gradients = -1 hence perp	M1 M1 C1	
	1(ii)	10	M1 A1	Use of pythagoras
	2	y = -4x + 19 cao Midpoint (4,3) verifying on line x + 2y = 10	M1 M1 A1 B1 C1	calculating m using (y -7) = m(x-3)
7	1	Cubic the correct way up -1, 2 and 5 indicated on x-axis 10 indicated on y-axis	G1 G1 G1	
	2	Negative quadratic curve Intercept (0,9) Through (3,0) and (-3,0)	G1 G1 G1	
	3	Any correct y value calculated (0,5), (1,1), (2,-1), (3,-1), (4,1) and (5,5) calculated Above points plotted Smooth parabola through the points	B1 B1 G1 G1	
8	1	$y = (x – 2)^2 - 4$	B2	M1 if y omitted, or for $y = (x + 2)^2 - 4$
	2(i)	Translation of $\begin{pmatrix}2\\0\end{pmatrix}$	B1 B1	
	2(ii)	y = f(x – 2)	B2	B1 for y = f(x + 2)
	3(i)	Translation of $\begin{pmatrix}-4\\0\end{pmatrix}$	B1 B1	
	3(ii)	sketch of parabola right way up min at (0,-4) and graph through (-2,0) and (2,0)	B1 B1	

9	1(i)	15.5	M1 A1	Use of Pythagoras
	1(ii)	x = 75.5°	M1 A1	(cos x = $\frac{4}{16}$) correct ratio and substitution
	2	√8 or 2√2 (but not ± √8)	M1 M1 A1	Use iof pythagoras use of tan Ɵ = opp / adj
	3	Smooth curve between y = 1 and y = -1 (90,0) and (270,0) (0,1), (180,-1), (360,1)	G1 G1 G1	
10	1(i)	9.0 or 8.96 or 8.960	M1 M1 A1	for use of cosine rule for square-rooting 'their' 80.2(8)
	1(ii)	13.3 or better (13.2577..)	M1 A1 A1	use of 'their' 0.5 x 4.1 x 6.6 x sin 108 correct values ans
	2	BC = 384 (or better) Total length = 1034m (or better)	M1 M1 A1 A1	recognisable attempt at cosine rule $BC^2 = 348^2 + 302^2 – 2x348x302xcos72$ BC = 383.86..... Total length = BC + 650 ft
11	1a)	$-6 \leq x \leq 6$	A1	
	1b)	$x \leq -\frac{5}{3}$, $x \geq \frac{5}{3}$	A1	
	1c)	$-\frac{10}{3} < x < 0$	A1	
	2	$x < -\frac{1}{2} and\ x > \frac{3}{2}$	M1 M1 M1 A1	Multiplying out denominators Forming a single quadratic 2 critical values
	3	$x < -\frac{4}{3}$, $x > 2$	M1 M1 A1	Factorising quadratic Critical values
12	1a)	(n + 1) and (n + 2)	A1	Both correct
	b)	= 3n + 3 = 3 (n + 1) 3 is a factor so the sum is a multiple of 3	M1 M1 A1	Adding expressions and simplifying result Factorising Conclusion with reason
	2	2n + 1 is an odd number $(2n + 1)^2 = 4n^2 + 4n + 1$ $4n^2 + 4n = 4 (n^2 + 1)$ = even so $4n^2 + 4n + 1$ is odd	M1 M1 A1	Expression for odd number Square expression Explanation

	3	$= \frac{4x(x^2+5)}{2(x^2+5)}$ $= \frac{4x}{2}$ $= 2x$ which is always even as is a multiple of 2	M1 M1 A1	Factorise Simplify Explanation
13	1(a)	-3h+2f+g	B1	OA - OB
	1(b)	$-\frac{1}{3}$ (6**h** + 2**f** + **g**) oe	M1 A1	PO = PA+AO
	2	BA = 3DC so lines are parallel	M1 M1 A1	Expression for BA Expression for DC Concluding statement
	3	QA and QM are both multiples of 2**j** – **k** so are parallel and have Q as a common point so are collinear	M1 M1 A1	QA = 1/3 (2**j** – **k**) oe QM = ½ (2**j** – **k**) oe Concluding statement
14	1	NP $\frac{3}{12}$ x $\frac{5}{11}$ = $\frac{5}{44}$ ND $\frac{3}{12}$ x $\frac{4}{11}$ = $\frac{1}{11}$ PN $\frac{5}{12}$ x $\frac{3}{11}$ = $\frac{5}{44}$ PD $\frac{5}{12}$ x $\frac{4}{11}$ = $\frac{5}{33}$ DN $\frac{4}{12}$ x $\frac{3}{11}$ = $\frac{1}{11}$ DP $\frac{4}{12}$ x $\frac{5}{11}$ = $\frac{5}{33}$ P(two different types) = $\frac{47}{66}$ Or (alternative solution) NN $\frac{3}{12}$ x $\frac{2}{11}$ = $\frac{1}{22}$	M1 M1 A1 M1	Multiplying each probability Adding their probabilities Correct solution Multiplying probability of same types

		PP $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ DD $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ P(two the same type) = $1 - \frac{19}{66}$ $= \frac{47}{66}$	M1 A1	Subtracting their answer from 1 Correct solution
	2	0.75 x 0.7 = 0.525 or 0.25 x 0.9 = 0.225 0.525 + 0.225 =0.75	M1 M1 A1	Multiplying probabilities for both situations Adding probabilities
	3a	$\frac{13664}{14345}$ or 0.95	A1	
	3b	$\frac{520}{12102} = \frac{260}{6051}$ or 0.04	A1	
	3c	$\frac{420}{13664} = \frac{15}{488}$ or 0.03	M1 A1	Allow M1 for $\frac{420}{14345} = \frac{84}{2869}$ or 0.03
15	1a	FD= 2.5 and 1.2 Plot on graph	M1 A1	FD = Frequency / class width
	1b	FD x class width 15 and 36	M1 A1	Both answers required
	2a	1.0, 3.7, 4.0, 1.5 Correct histogram drawn	A1 A1 M1 A1	Frequency density calculated 1 mark for 3 correct Mark awarded for 2 correct bars All correct
	2b	300 – 240 = 60 mins 90	M1 A1	Calculation to find the class width

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Diagnosis Worked Solutions

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1 Solving quadratic equations

Question 1

Solve $x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or -4

(2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0 \rightarrow y = 3$ or $y = 4$

$x^4 - 7x^2 + 12 = 0 \rightarrow$ Let $x^2 = y$

$(x^2)^2 - 7x^2 + 12 = 0 \rightarrow y^2 - 7y + 12 = 0 \rightarrow y = 3$ or $y = 4$

$\rightarrow x^2 = 3$ or $x^2 = 4$

$\rightarrow x = \pm\sqrt{3}$ or $x = \pm 2$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

$x^2 - 6x + 2 = (x-3)^2 - 9 + 2$

$= (x-3)^2 - 7$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

Minimum point of $x^2 - 6x + 2$ is therefore $(3, -7)$

(1)

Total / 10

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2 Changing the subject

Question 1

Make v the subject of the formula $E = \frac{1}{2}mv^2$

$$E = \frac{1}{2}mv^2$$
$$\Rightarrow 2E = mv^2$$
$$\Rightarrow \frac{2E}{m} = v^2$$
$$\pm\sqrt{\frac{2E}{m}} = v$$

(3)

Question 2

Make r the subject of the formula $V = \frac{4}{3}\pi r^2$

$$V = \frac{4}{3}\pi r^3$$ get rid of fractions
$$3V = 4\pi r^3$$ make r^3 the subject.
$$\frac{3V}{4\pi} = r^3$$ cube both sides
$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

(3)

Question 3

Make c the subject of the formula $P = \frac{C}{C+4}$

$$P = \frac{C}{C+4}$$ Get rid of fractions
$$\Rightarrow P(C+4) = C$$ Expand brackets
$$\Rightarrow PC + 4P = C$$ Get terms with C on L.H.S., other terms on R.H.S.
$$PC + 4P - C = 0$$
$$PC - C = -4P$$ Factorise.
$$c(P-1) = -4P$$ Divide
$$C = \frac{-4P}{P-1} \quad \left(= \frac{4P}{1-P}\right)$$

(4)

Total / 10

3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines y = 3x + 1 and x + 3y = 6

$y = 3x + 1$ and $x + 3y = 6$

$x + 3(3x+1) = 6$

$x + 9x + 3 = 6$

$10x = 3$

$x = \frac{3}{10}$

$y = 3\left(\frac{3}{10}\right) + 1$

$= \frac{9}{10} + 1$

$= 1\frac{9}{10}$

(3)

$\left(\frac{3}{10}, 1\frac{9}{10}\right)$ or $(0.3, 1.9)$

(3)

Question 2

Find the coordinates of the point of intersection of the lines 5x + 2y = 20 and y = 5 – x

(ii) $5x + 2y = 20$ & $y = 5 - x$.

Solving simultaneously

$5x + 2(5-x) = 20$ M1

$\Rightarrow 5x + 10 - 2x = 20$

$\Rightarrow 3x = 10$

$\Rightarrow x = 10/3$, $y = 5 - 10/3 = 5/3$

Pt. of intersection is $(10/3, 5/3)$ A2

Note – if you round these fractions to decimals (10.3, 1.7) you lose a mark.

(3)

Question 3

Solve the simultaneous equations

$x^2 + y^2 = 5$ $y = 3x + 1$

Sub in $y = 3x+1$ into equation 2.

$x^2 + (3x+1)^2 = 5$

$x^2 + (3x+1)(3x+1) = 5$

$x^2 + 9x^2 + 3x + 3x + 1 = 5$

$10x^2 + 6x + 1 = 5$

$10x^2 + 6x - 4 = 0$

$(\div 2)$

$5x^2 + 3x - 2 = 0$

$(5x-2)(x+1) = 0$

$x = \frac{2}{5}$ or $x = -1$

when $x = \frac{2}{5}$

$y = \left(3 \times \frac{2}{5}\right) + 1$

$= \frac{6}{5} + \frac{5}{5} = \frac{11}{5}$

when $x = -1$

$y = (3 \times -1) + 1$

$= -3 + 1$

$= -2$

(4)

Total / 10

4 Surds

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

$$(3+\sqrt{2})(3-\sqrt{2})$$
$$= 3^2 + 3\sqrt{2} - 3\sqrt{2} - (\sqrt{2})^2$$
$$= 9 - 2$$
$$= 7$$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

$$\frac{(1+\sqrt{2})}{(3-\sqrt{2})} = \frac{(1+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$
$$= \frac{3+\sqrt{2}+3\sqrt{2}+(\sqrt{2})^2}{7}$$
$$= \frac{3+4\sqrt{2}+2}{7}$$
$$= \frac{5}{7} + \frac{4}{7}\sqrt{2}$$

To rationalise a denominator of form $(x+\sqrt{y})$ multiply top + bottom by $(x-\sqrt{y})$

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

(i) $5\sqrt{8} + 4\sqrt{50}$
$= 5\sqrt{4}\sqrt{2} + 4\sqrt{25}\sqrt{2}$
$= 5\times 2\sqrt{2} + 4\times 5\sqrt{2}$
$= 10\sqrt{2} + 20\sqrt{2}$
$= 30\sqrt{2}$

Always look for square number factors

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

$$\frac{\sqrt{3}}{6-\sqrt{3}} = \frac{\sqrt{3}}{6-\sqrt{3}} \times \frac{(6+\sqrt{3})}{(6+\sqrt{3})}$$
$$= \frac{\sqrt{3}\times 6 + \sqrt{3}\sqrt{3}}{6^2 - (\sqrt{3})^2}$$
$$= \frac{6\sqrt{3} + 3}{36-3}$$
$$= \frac{3+6\sqrt{3}}{33}$$
$$= \frac{3}{33} + \frac{6}{33}\sqrt{3}$$
$$= \frac{1}{11} + \frac{2}{11}\sqrt{3}.$$

(3)

Total / 10

5 Indices

Question 1

Simplify the following

(i) a^0 (1)

(ii) $a^6 \div a^{-2}$ (1)

(iii) $(9a^6b^2)^{-0.5}$ (3)

(i) $a^0 = 1$

(ii) $a^6 \div a^{-2} = a^{6--2}$

$= a^8$

(iii) $(9a^6b^2)^{-1/2} = (3^2a^6b^2)^{-1/2}$

$= 3^{-1}a^{-3}b^{-1}$ $\left(= \frac{1}{3a^3b} \right)$

Question 2

(i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$ (2)

(ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ (3)

i) $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = (25)^{\frac{1}{2}} = \sqrt{25} = \pm 5$

ii) $\frac{(2x^2y^3z)^5}{4y^2z} = \frac{2^5x^{10}y^{15}z^5}{2^2y^2z^1}$

$= 2^{5-2}x^{10}y^{15-2}z^{5-1}$

$= 2^3x^{10}y^{13}z^4 = 8x^{10}y^{13}z^4$

Total / 10

6 Properties of Lines

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

Grad of AB = $\frac{9-2}{7-0}$ = 1

Grad of BC = $\frac{10-9}{6-7}$ = -1

Product of gradients = 1 x -1 = -1 → AB and BC perpendicular

(3)

(ii) Find the length of AC

$(6-0)^2 + (10-2)^2 = AC^2$
AC = 10

(2)

Question 2

Find, in the form y = mx + c, the equation of the line passing through A (3,7) and B (5,-1).
Show that the midpoint of AB lies on the line x + 2y = 10

$m = \frac{-1-7}{5-3} = \frac{-8}{2} = -4$

$y = -4x + c$

Substitute in (3,7) [5,-1] would do equally as well

$7 = -4 \times 3 + c$
$\Rightarrow 19 = c$

$\Rightarrow y = -4x + 19$

Midpoint of AB = (4, 3)

Sub. in to $x + 2y = 10$ & show that equation is true.

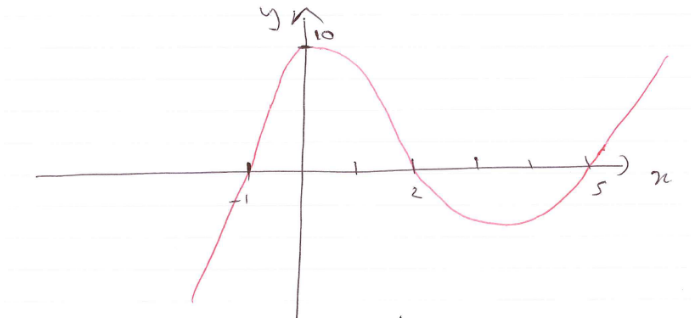
$4 + 2 \times 3 = 4 + 6 = 10$ TRUE.

(5)

Total / 10

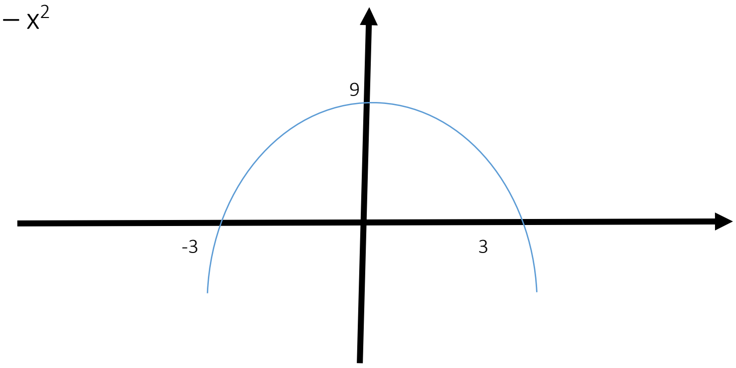
7 Sketching curves

Question 1
In the cubic polynomial f(x), the coefficient of x^3 is 1. The roots of f(x) = 0 are -1, 2 and 5.
Sketch the graph of y = f(x)



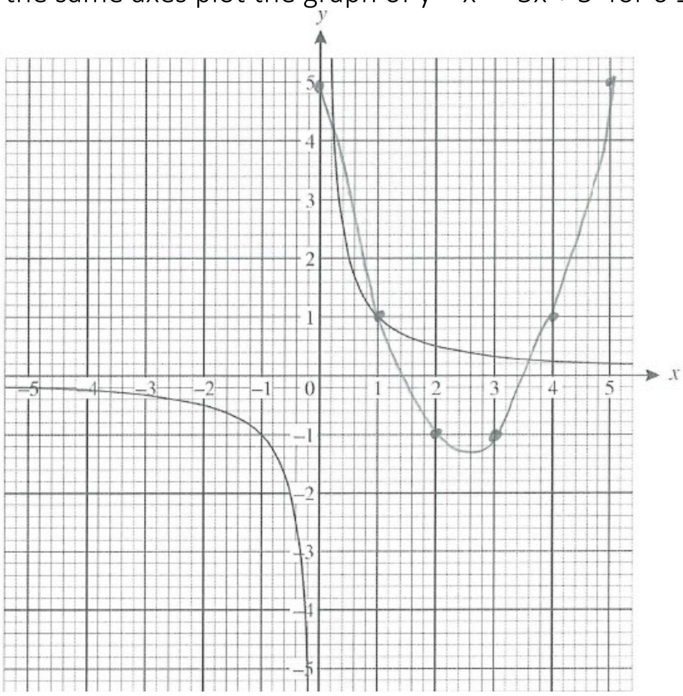
(3)

Question 2
Sketch the graph of $y = 9 - x^2$



(3)

Question 3
The graph below shows the graph of $y = \frac{1}{x}$
On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \le x \le 5$



x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	-25
$+5$	+5	+5	+5	+5	+5	+5
y	5	1	-1	-1	1	5

(4)

Total / 10

8 **Transformation of functions**

Question 1

The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

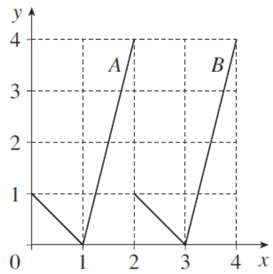
Write down an equation for the translated curve. You need not simplify your answer.

$y = (x-2)^2 - 4$

(2)

Question 2

This diagram shows graphs A and B.



(i) State the transformation which maps graph A onto graph B

A movement of 2 to the right is a translation of $\begin{pmatrix} +2 \\ 0 \end{pmatrix}$

(2)

(ii) The equation of graph A is $y = f(x)$.
Which one of the following is the equation of graph B ?

$y = f(x) + 2$	$y = f(x) - 2$	$y = f(x+2)$	$y = f(x-2)$
$y = 2f(x)$	$y = f(x+3)$	$y = f(x-3)$	$y = 3f(x)$

$f(x-2)$

$f(x) + 2$

up 2

$f(x+2) \xleftarrow{2} f(x) \xrightarrow{2} f(x-2)$

down 2

$f(x) - 2$

Answer f(x-2)

(2)

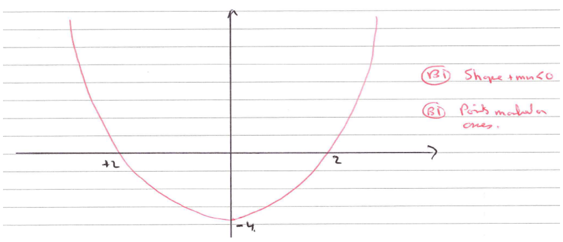
Question 3

(i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

- Translation B1
- $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ B1 (or 4 units to the left)

(2)

(ii) Sketch the graph of $y = x^2 - 4$



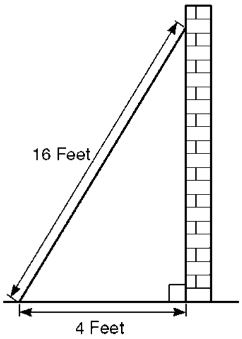
(2)

Total / 10

9 Trigonometric ratios

Question 1

Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall.
The ladder is 16 feet long.



The foot of the ladder is 4 feet from the base of the wall.

(i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

$\sqrt{16^2 - 4^2}$

$\sqrt{256 - 16}$ correct substitution (M1)

$\sqrt{240}$

15.49

15.5 (3sf) (A1) (2)

(ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 sig fig

$\cos x = \frac{4}{16}$ correct ratio and substitution (M1)

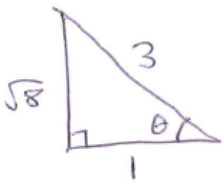
$\cos x = 0.25$

$x = 75.522$

$x = 75.5°$ (A1) (2)

Question 2

Given that $\cos \theta = \frac{1}{3}$ and θ is acute, find the exact value of $\tan \theta$

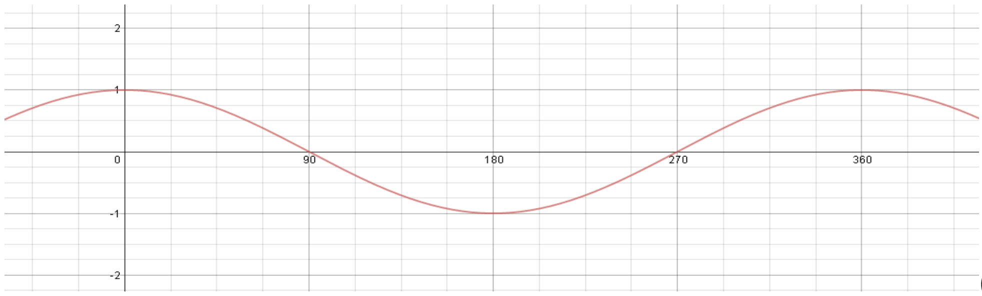


$\tan \theta = \frac{\text{opp.}}{\text{Adj}} = \frac{\sqrt{8}}{1} = \sqrt{8}$

(3)

Question 3

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360°$

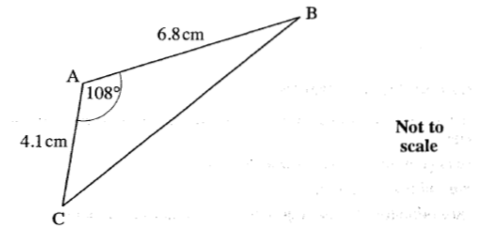


(3)

Total / 10

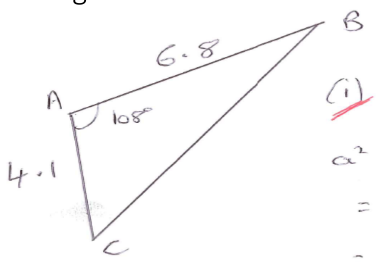
10 Sine / Cosine Rule

Question 1



For triangle ABC, calculate

(i) the length of BC



(i) By the Cosine Rule.

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 6.8^2 + 4.1^2 - 2 \times 6.8 \times 4.1 \times \cos 108$

$= 63.05 - - 17.23$

$= 80.28$

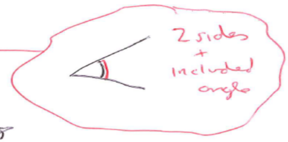
$\Rightarrow a = \sqrt{80.28} = 8.960$

(3)

(ii) the area of triangle ABC

Area of a Triangle

$= \frac{1}{2} ab \sin C$



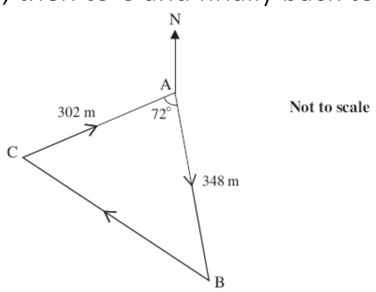
$= \frac{1}{2} \times 4.1 \times 6.8 \times \sin 108$

$= 13.26$

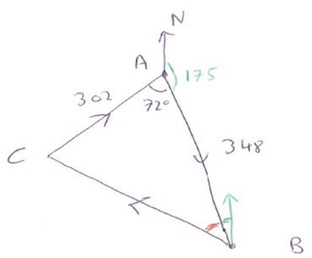
(3)

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Calculate the total length of the course for this race.



Use the Cosine Rule to find CB

$CB^2 = 302^2 + 348^2 - 2 \times 302 \times 348 \times \cos 72$

$CB = 384$

Total length = 384 + 650 = 1034m

(4)

Total / 10

11 <u>Inequalities</u>

Question 1

Solve

a) $x^2 - 36 \le 0$

$$(x+6)(x-6) \le 0$$
$$-6 \le x \le 6 \qquad \text{(A1)}$$

b) $9x^2 - 25 \ge 0$

$$(3x-5)(3x+5) \ge 0$$
$$x \le -\frac{5}{3},\ x \ge \frac{5}{3} \qquad \text{(A1)}$$

c) $3x^2 + 10x < 0$

$$x(3x+10) < 0$$
$$-\frac{10}{3} < x < 0 \qquad \text{(A1)}$$

(3)

Question 2

Solve $\frac{21}{x+2} - \frac{5}{x+1} < 4$

$21(x+1) - 5(x+2) < 4(x+2)(x+1)$ (M1)

$21x + 21 - 5x - 10 < 4(x^2 + 3x + 2)$

$16x + 11 < 4x^2 + 12x + 8$

$0 < 4x^2 - 4x - 3$ (M1)

$0 < (2x+1)(2x-3)$

Critical values $x = -\frac{1}{2}$ or $x = \frac{3}{2}$ (M1)

$x < -\frac{1}{2}$ and $x > \frac{3}{2}$ (A1)

....

(4)

Question 3

Solve $3x^2 - 8 > 2x$

$3x^2 - 2x - 8 > 0$

$(3x+4)(x-2) > 0$ (M1)

Critical values x = -4/3 and x=2 (M1)

$x < -\frac{4}{3},\ x > 2$ (A1)

....

(3)

Total / 10

12 Algebraic proof

Question 1

a) If n is a positive integer, write down expressions for the next two consecutive integers.

(n + 1) and (n + 2) 1M both correct

(1)

b) Use algebra to prove that the sum of three positive consecutive integers is always a multiple of 3.

n + n + 1 + n + 2
= 3n + 3
= 3 (n + 1)
3 is a factor so the sum is a multiple of 3

- Adding expressions and simplifying result
- Factorising
- Conclusion with reason

(3)

Question 2

Prove that the square of an odd number is also odd.

2n is an even number then 2n + 1 is an odd number

$(2n + 1)^2 = 4n^2 + 4n + 1$

$4n^2 + 4n = 4 (n^2 + 1)$ so this expression is a multiple of 4 hence even

so $4n^2 + 4n + 1$ is odd

- Writing algebraic expression for odd number
- Squaring expression
- Explain why result is odd

(3)

Question 3

Given that x is a positive integer, prove that $\frac{4x^3+20x}{2x^2+10}$ is always even.

$= \frac{4x(x^2+5)}{2(x^2+5)}$

$= \frac{4x}{2}$

$= 2x$ which is always even as is a multiple of 2

- Factorise
- Simplify
- Explain why result is even

(3)

Total / 10

13 Vectors

Question 1

OAP is a triangle

$\overrightarrow{OA} = 2\mathbf{f} + \mathbf{g}$ and $\overrightarrow{OB} = 3\mathbf{h}$

P is the point on AB such that AP: PB = 2:1

(a) Find the vector $\overrightarrow{BA}$ in terms of **f, g** and **h.**

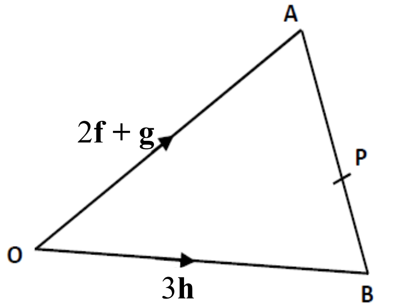


Diagram NOT drawn accurately

(1)

(b) Find the vector $\overrightarrow{PO}$ in terms of **f, g** and **h**

$\overrightarrow{PO} = \overrightarrow{PA} + \overrightarrow{AO} = \frac{2}{3}(-3\mathbf{h} + 2\mathbf{f} + \mathbf{g}) - (2\mathbf{f} + \mathbf{g})$ (M1)

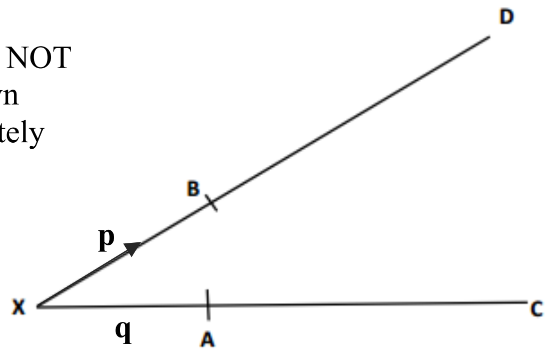
$\overrightarrow{PO} = -2\mathbf{h} - \frac{2}{3}\mathbf{f} - \frac{1}{3}\mathbf{g}$ or $-\frac{1}{3}(6\mathbf{h} + 2\mathbf{f} + \mathbf{g})$ oe simplified expression (A1)

....

(2)

Question 2

Diagram NOT drawn accurately



B is the point on AD such that XB:BD is

A is the point on XC such that XA:XC is 1:2

$\overrightarrow{XB} = \mathbf{p}$ and $\overrightarrow{XA} = \mathbf{q}$

Use vectors to explain the geometrical relationships between the line segments BA and DC.

$\overrightarrow{BA} = -p + q$ (M1)

$\overrightarrow{DC} = -3p + 3q$ (M1)

$\overrightarrow{BA} = 3\overrightarrow{DC}$ so the lines are parallel (A1) and DC is 3 times the length of BA (A1)

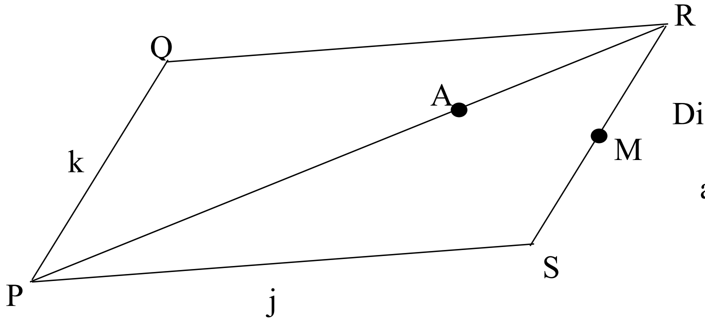
(4)

Question 3

PQRS is a parallelogram.

A is the point on PR such that PA:AR is 2:1

M is the midpoint of RS.



(b) Prove that Q, A and M are co-linear.

$\overrightarrow{QA} = -\mathbf{k} + 2/3(\mathbf{k} + \mathbf{j}) = -1/3\mathbf{k} + 2/3\mathbf{j} = 1/3\ (2\mathbf{j} - \mathbf{k})$ (M1) accept any equivalent vector

$\overrightarrow{QM} = -\mathbf{k} + \mathbf{j} + \frac{1}{2}\mathbf{k} = -1/2\ \mathbf{k} + \mathbf{j} = \frac{1}{2}(2\mathbf{j} - \mathbf{k})$ (M1) accept any equivalent vector

$\overrightarrow{QA}$ and $\overrightarrow{QM}$ are both multiples of $2\mathbf{j} - \mathbf{k}$ so are parallel and have Q as a common point so are collinear

....

(3)

Total / 10

14 Probability

Question 1

A box contains 3 new batteries, 5 partly used batteries and 4 dead batteries.

Kelly takes two batteries at random.

Work out the probability that she picks two different types of batteries.

NP $\frac{3}{12} \times \frac{5}{11} = \frac{5}{44}$

ND $\frac{3}{12} \times \frac{4}{11} = \frac{1}{11}$

PN $\frac{5}{12} \times \frac{3}{11} = \frac{5}{44}$

PD $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ Multiplying each probability M1

DN $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ Adding their probabilities M1

DP $\frac{4}{12} \times \frac{5}{11} = \frac{5}{33}$ Correct solution A1

P(two different types) = $\frac{47}{66}$

Or

NN $\frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

PP $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ Multiplying probability of MTS by 6 M1

DD $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ Subtracting their answer from 1 M1

Correct solution A1

P(two the same type) = $1 - \frac{19}{66} = \frac{47}{66}$

....

(3)

Question 2

Caleb either walks to school or travels by bus.

The probability that he walks to school is 0.75.

If he walks to school, the probability that he will be late is 0.3.

If he travels to school by bus, the probability that he will be late is 0.1.

Work out the probability that he will not be late.

0.75 x 0.7 = 0.525 or 0.25 x 0.9 = 0.225 M1

0.525 + 0.225 = M1

0.75 A1

....

(3)

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Question 3

'he two way table shows the number of deaths and serious injuries caused by road traffic accidents in Great Britain in 2013.

		Speed Limit			
		20 mph	**30 mph**	**40 mph**	**Total**
Type of Injury	**Fatal**	6	520	155	681
	Serious	420	11582	1662	13664
	Total	426	12102	1817	14345

Work out an estimate for the probability:

(a) that the accident is serious.

$\frac{13664}{14345}$ or 0.95 A1 ....

(1)

(b) that the accident is fatal given that the speed limit is 30 mph.

$\frac{520}{12102} = \frac{260}{6051}$ or 0.04

....

(1)

(c) that the accident happens at 20 mph given that the accident is serious.

$\frac{420}{13664} = \frac{15}{488}$ or 0.03 M2 (Correct working must be seen)

Allow M1 for $\frac{420}{14345} = \frac{84}{2869}$ or 0.03

....

(2)

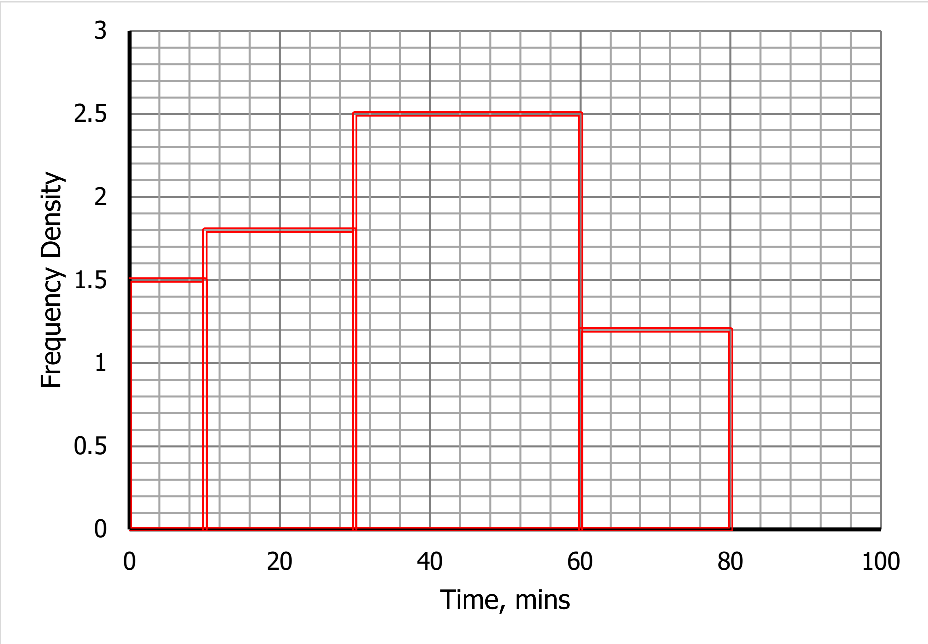
Total / 10

15 Statistics

Question 1

The histogram and the frequency table show some information about how much time vehicles spent in a car park.

Time, minutes	Frequency	Class Width	Freq. Density
$0 < x \le 10$	15	10	1.5
$10 < x \le 30$	36	20	1.8
$30 < x \le 60$	75	30	2.5
$60 < x \le 80$	24	20	1.2
Total	150		



a) Use the information to complete the histogram

(2)

b) Use the histogram to find the missing frequencies in the table

1.5 x 10 = 15 B1

1.8 x 20 = 36 B1

.........15 and 36 ....

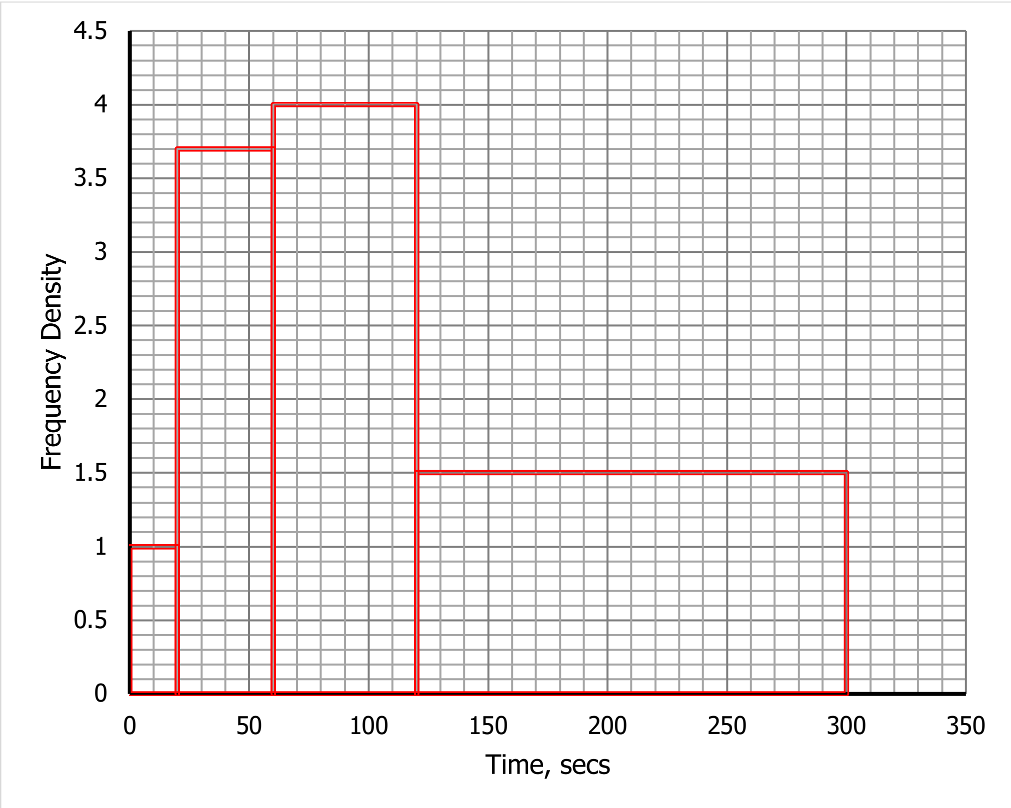
(2)

Question 2

The table shows the length of 678 phone calls made at a call centre

Time, secs	Frequency	Class Width	Freq. Density
$0 < x \leq 20$	20	20	1.0
$20 < x \leq 60$	148	40	3.7
$60 < x \leq 120$	240	60	4.0
$120 < x \leq 300$	270	180	1.5
Total	678		

a) Draw a fully labelled histogram to show the length of the phone calls.



(4)

b) Estimate the number of phone calls that lasted more than 4 minutes.

4 minutes = 4 x 60 secs = 240 secs

300 – 240 = 60 mins

60 x 1.5 = (90 calls) M1

............90 calls...............A1...............

(2)

Total / 10

The Bridge to A level

Therapy

Therapy for Topics

All therapy references are referenced to the ***PiXL Maths App*** or ***MyMaths***

For the ***PiXL Maths App*** you need to navigate to the ***Therapy*** videos as follows:

After logging in, select the ***Design a Test*** tab.

This will give you 7 tabs to choose from (Number, Algebra, Ratio & Proportion, Geometry, Probability, Statistics and Problem Solving). Select one of these; this is the ***Topic***.

You will now see a menu of ***Test Titles***. Select the one you need, and you will need to select the ***Begin Test*** button.

<u>You need not do this test,</u> but it will bring up the ***Therapy Video*** button which you now select.

This will give you the menu of Therapy video titles applicable to this test, Select the one you need.

<u>In Summary</u>, all therapy references are for therapy videos from the PiXL Maths App.
The relevant video is found by accessing

Take a Test / Topic / Test Title / Therapy video title

The PiXL Maths App can be downloaded free from the PiXL website

http://mathsapp.pixl.org.uk/

School id : CL4199
User id : SURNAMEFIRST INITIAL (eg. WILESL)
Password : 15FIRSTINITIALSURNAME (e.g 15LWILES)

For ***MyMaths*** your school login details are :
School user name: Leysland
School password : Octagon

The hyperlinks in this document will take you directly to the pages you will need. (You may need to copy and paste the link into your web browser).

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Therapy for Topic 1 Quadratic equations
Skye delete line above and change this to Therapy Quadratic equations at title and repeat for each below

PiXL Therapy - Rearranging and solving quadratics, Completing the square, Factorising, Formula

Maths App Reference
1. Algebra / Graph transformations part a / Calculate the minimum point of a quadratic function
2. Algebra / Quadratics / Solve a quadratic by factorising where a is not 1 **and** solve a quadratic by completing the square

MyMaths Reference
https://app.mymaths.co.uk/1784-resource/quadratic-equations-1
https://app.mymaths.co.uk/192-resource/quadratic-equations-2
https://app.mymaths.co.uk/193-resource/completing-the-square
https://app.mymaths.co.uk/194-resource/quadratic-formula

Therapy for Topic 2 Algebra

Maths App Reference
1. Algebra / Harder equations and re-arranging formulae / Re-arrange a formula where the subject appears more than once

MyMaths Reference
https://app.mymaths.co.uk/175-resource/factorising-quadratics-2
https://app.mymaths.co.uk/207-resource/rearranging-2
https://app.mymaths.co.uk/176-resource/cancelling-algebraic-fractions
https://app.mymaths.co.uk/177-resource/adding-algebraic-fractions
https://app.mymaths.co.uk/178-resource/multiplying-algebraic-fractions

Therapy for Topic 3 Simultaneous equations

PiXL Therapy - Linear, Non-linear

Maths App Reference
1. Algebra / Simultaneous equations / Solve linear and non-linear simultaneous equations

MyMaths Reference
https://app.mymaths.co.uk/198-resource/simultaneous-equations-3
https://app.mymaths.co.uk/195-resource/quadratic-simultaneous-equs

Therapy for Topic 4 Surds

PiXL Therapy - Surds

Maths App Reference
1. Number / Surds / Simplify a surd
2. Number / Surds / Rationalise a denominator
3. Number / Surds / Operate with surds

MyMaths Reference
https://app.mymaths.co.uk/599-resource/surds-part-1
https://app.mymaths.co.uk/600-resource/surds-part-2

Therapy for Topic 5 Indices

PiXL Therapy - Negative and fractional index laws

Maths App Reference
1. Number / Indices / Evaluate positive, negative and fractional indices

MyMaths Reference
https://app.mymaths.co.uk/597-resource/indices-part-2
https://app.mymaths.co.uk/598-resource/indices-part-3

Therapy for Topic 6 Properties of lines

MyMaths Reference
https://app.mymaths.co.uk/559-resource/equation-of-a-line
https://app.mymaths.co.uk/3270-resource/equation-of-a-line-3

Therapy for Topic 7 Sketching curves

PiXL Therapy - Turning points and completing the square

MyMaths Reference
https://app.mymaths.co.uk/226-resource/sketching-quadratic-graphs-2
https://app.mymaths.co.uk/3266-resource/sketching-cubic-graphs
https://app.mymaths.co.uk/588-resource/sketching-polynomials

Therapy for Topic 8 Transformation of functions

PiXL Therapy - Translate a function

Maths App Reference
1. Algebra/ Graph transformations part a / Recognise the shape of quadratic graph transformations
2. Algebra / Graph transformations part a / Calculate the minimum point of a quadratic function
3. Algebra / Graph transformations part b / Recognise trigonometric graph transformations

4. Algebra / Graph transformations part b / Interpret values from a transformed trigonometric graph

MyMaths Reference
https://app.mymaths.co.uk/585-resource/transforming-graphs
https://app.mymaths.co.uk/3265-resource/transforming-graphs-1
https://app.mymaths.co.uk/229-resource/transforming-graphs-2

Therapy for Topic 9 Pythagoras' Theorem and Trigonometric ratios

Maths App Reference
1. Geometry / Trigonometry / Calculate an unknown angle or side using SOH CAH TOA
2. Geometry / Trigonometry / Apply Pythagoras' Theorem and SOH CAH TOA in 3D contexts
3. Geometry / Pythagoras' Theorem / Calculate the length of a line in 2D or 3D problems **and** Calculate the length of a line using Pythagoras from coordinates

MyMaths Reference
https://app.mymaths.co.uk/300-resource/pythagoras-theorem
https://app.mymaths.co.uk/301-resource/pythagoras-3d
https://app.mymaths.co.uk/321-resource/trig-missing-angles
https://app.mymaths.co.uk/322-resource/trig-missing-sides
https://app.mymaths.co.uk/328-resource/3d-trigonometry

Therapy for Topic 10 Sine / Cosine Rule

PiXL Therapy - Cosine rule, Sine rule

Maths App Reference
1. Geometry / Sine and Cosine Rules / Apply the sine and cosine rules to calculate a length or an angle
2. Geometry / Sine and Cosine Rules / Calculate the area of non-right angled triangles and of a segment.

MyMaths Reference
https://app.mymaths.co.uk/325-resource/cosine-rule-sides
https://app.mymaths.co.uk/326-resource/cosine-rule-angles
https://app.mymaths.co.uk/324-resource/sine-rule
https://app.mymaths.co.uk/327-resource/trig-area-of-a-triangle

Therapy for Topic 11 Inequalities

PiXL Therapy - Quadratic inequalities

Maths App Reference
1. Algebra / Inequalities / Solve inequalities
MyMaths Reference

https://app.mymaths.co.uk/232-resource/inequations
https://app.mymaths.co.uk/235-resource/quadratic-inequalities

Therapy for Topic 12 Algebraic Proof

PiXL Therapy - Proof

MyMaths Reference
https://app.mymaths.co.uk/1781-resource/identities
https://app.mymaths.co.uk/1777-resource/proof

Therapy for Topic 13 Vectors

PiXL Therapy - Vectors

Maths App Reference

1. Geometry / Vectors / Construct a resultant vector from a diagram
2. Geometry / Vectors / Recognise when 3 or more points are collinear
3. Geometry / Vectors / Perform vector arithmetic

MyMaths Reference
https://app.mymaths.co.uk/332-resource/vectors-1
https://app.mymaths.co.uk/333-resource/vectors-2

Therapy for Topic 14 Probability

PiXL Therapy - Conditional probability

Maths App Reference

1. Probability / Dependent events / Calculate probabilities from tree diagrams using AND / OR statements
2. Probability / Venn diagrams – part b / Calculate probability from Venn diagrams and set notation

MyMaths Reference
https://app.mymaths.co.uk/1730-resource/venn-diagrams-2
https://app.mymaths.co.uk/381-resource/probability-revision
https://app.mymaths.co.uk/382-resource/the-or-rule
https://app.mymaths.co.uk/383-resource/independent-probability

Therapy for Topic 15 Statistics: Averages and Graphs

PiXL Therapy - Histograms

Maths App Reference

1. Statistics / Histogram tables / Calculating the frequency density
2. Statistics / Interpreting histograms / Calculating the frequency from a histogram
3. Statistics / Averages from a frequency table / Calculate the mean from a grouped frequency table
4. Statistics / Frequency polygon graphs / Interpret values from a frequency polygon graph
5. Statistics / Cumulative frequency graphs / Interpret a cumulative frequency diagram
6. Statistics / Boxplots / Interpret a boxplot

MyMaths Reference

https://app.mymaths.co.uk/365-resource/histograms
https://app.mymaths.co.uk/1739-resource/frequency-polygons
https://app.mymaths.co.uk/362-resource/cumulative-frequency-1
https://app.mymaths.co.uk/363-resource/cumulative-frequency-2
https://app.mymaths.co.uk/364-resource/box-and-whisker-plots

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The Bridge to A level

Test yourself

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1 Solving quadratic equations

Question 1

Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2

(4)

Question 2

(i) Write $4x^2 - 24x + 27$ in the form of $a(x - b)^2 + c$

(4)

(ii) State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$.

(2)

Total / 10

2 Changing the Subject

Question 1

Make t the subject of the formula $s = \frac{1}{2}at^2$

(3)

Question 2

Make x the subject of $3x - 5y = y - mx$

(3)

Question 3

Make x the subject of the equation $y = \frac{x+3}{x-2}$

(4)

Total / 10

3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines $x + 2y = 5$ and $y = 5x - 1$

(3)

Question 2

The lines $y = 5x - a$ and $y = 2x + 18$ meet at the point $(7, b)$.

Find the values of a and b.

(3)

Question 3

A line and a curve has the following equations :

$$3x + 2y = 7 \qquad\qquad y = x^2 - 2x + 3$$

Find the coordinates of the points of intersection of the line and the curve by solving these simultaneous equations algebraically

(4)

Total / 10

4 <u>Surds</u>

Question 1

(i) Simplify $\sqrt{24} + \sqrt{6}$

(2)

(ii) Express $\frac{36}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

(3)

Question 2

(i) Simplify $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

(2)

(ii) Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

Total / 10

5 Indices

Question 1

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$

(2)

(ii) $16^{\frac{3}{4}}$

(2)

Question 2

(i) Find a, given that $a^3 = 64x^{12}y^3$

(2)

(ii) $\left(\frac{1}{2}\right)^{-5}$

(2)

Question 3

Simplify $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

(2)

Total / 10

6 Properties of Lines

Question 1

The points A (-1,6), B (1,0) and C (13,4) are joined by straight lines. Prove that AB and BC are perpendicular.

(2)

Question 2

A and B are points with coordinates (-1,4) and (7,8) respectively. Find the coordinates of the midpoint, M, of AB.

(1)

Question 3

A line has gradient -4 and passes through the point (2,-6). Find the coordinates of its points of intersection with the axes.

(4)

Question 4

Find the equation of the line which is parallel to $y = 3x + 1$ and which passes through the point with coordinates (4,5).

(3)

Total / 10

7 Sketching curves

Question 1

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$

Sketch the graph of $y = f(x)$

(3)

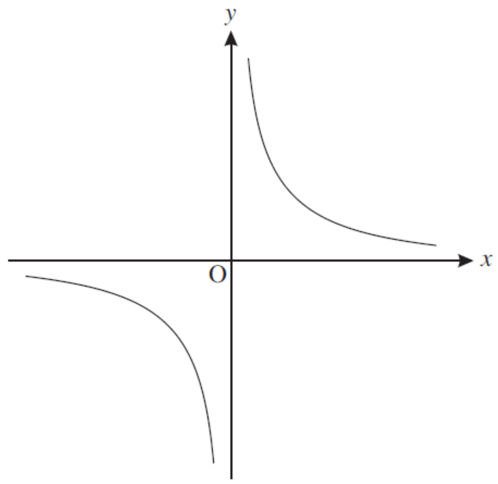
Question 2

Sketch the graph of $y = x(x - 3)^2$

(3)

Question 3

This diagram shows a sketch of the graph of $y = \frac{1}{x}$



Sketch the graph of $y = \frac{1}{x-2}$, showing clearly any points where it crosses the axes.

(3)

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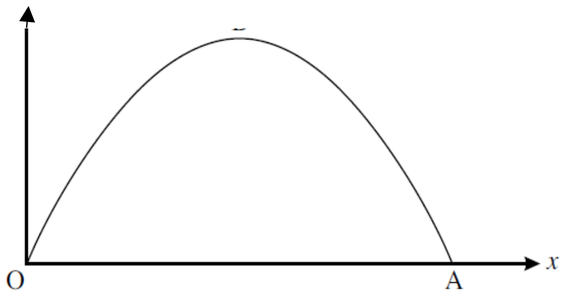
Question 4

This curve has equation $y = \frac{1}{5}x(10 - x)$. State the value of x at the point A.



(1)

Total / 10

8 Transformation of functions

Question 1

The graph of $y = x^2 - 8x + 25$ is translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$. State an equation for the resultant graph.

(1)

Question 2

$f(x) = x^3 - 5x + 2$

Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$

(4)

Question 3

You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$

Show that $f(x - 4) = 2x^3 - 17x^2 + 33x$

(3)

Question 4

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$.

The graph of $y = f(x)$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

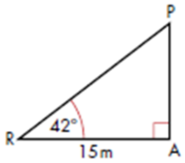
(2)

Total / 10

9 Trigonometric ratios

Question 1

AP is a telephone pole. The angle of elevation of the top of the pole from the point R on the ground is 42°as seen in the diagram.



Calculate the height of the pole. Give your answer to 3 significant figures.

(3)

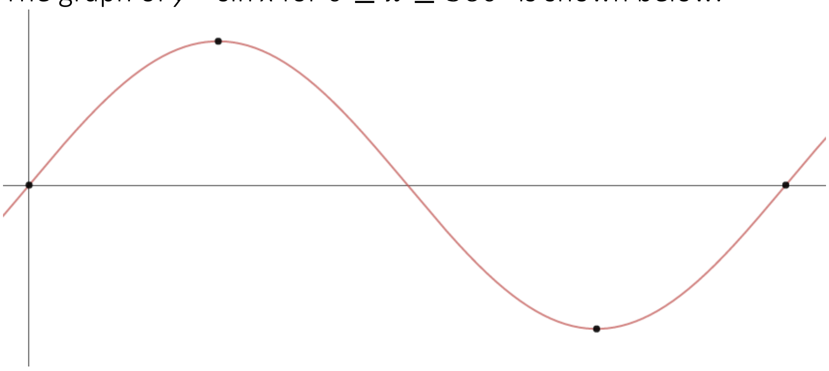
Question 2

Given that sin Θ = $\frac{\sqrt{3}}{4}$, find in surd form the possible values of cos Θ.

(3)

Question 3

The graph of y = sin x for $0 \leq x \leq 360°$ is shown below.



What are the coordinates of the 4 points labelled on the graph?

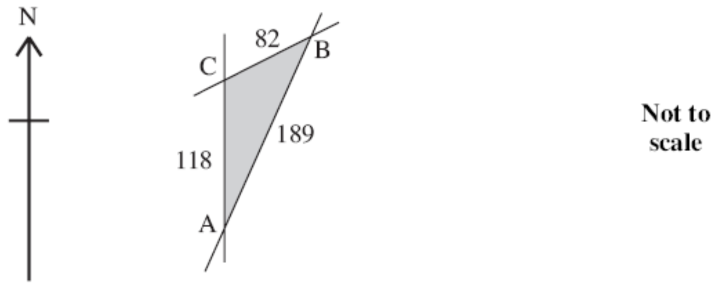
(………, ………)
(………, ………)
(………, ………)
(………, ………)
(4)

Total / 10

10 Sine / Cosine Rule

Question 1

This diagram shows a village green which is bordered by 3 straight roads AB, BC and AC. The road AC runs due North and the measurements are shown in metres.



(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°

(4)

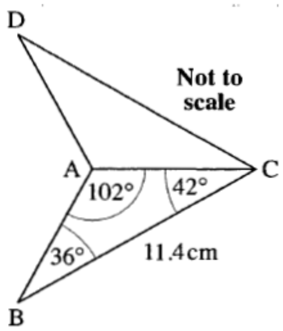
(ii) Calculate the area of the village green.

(2)

Question 2

This diagram shows a logo ABCD. It is symmetrical about AC.

Find the length of AB and hence find the area of the logo



(4)

Total / 10

11 ## Inequalities

Question 1

Solve the inequality $\boldsymbol{x^2 < 3(x+6)}$

(3)

Question 2

Solve the inequality $\boldsymbol{x^2 > 3x + 4}$

(3)

Question 3

A rectangle has length 3x cm and width (x+2) cm. The area of the rectangle is less than 90cm. Find the possible range of values for x.

(4)

Total / 10

12 Algebraic proof

Question 1

a) If n is a positive integer explain why the expression 2n + 1 is always an odd number.

(1)

b) Use algebra to prove that the product of two odd numbers is also odd.

(4)

Question 2

a) Prove that the sum of four consecutive whole numbers is always even.

(3)

b) Give an example to show that the sum of four consecutive number is not always divisible by 4.

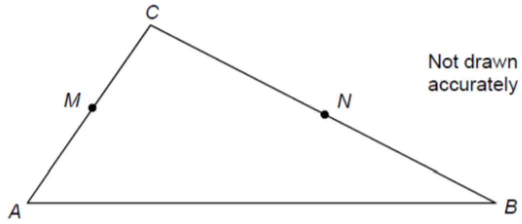
(2)

Total / 10

13 Vectors

Question 1

Triangle ABC has points M as the midpoints of AC
and point N such that BN:CN = 2:3



$\overrightarrow{AM} = a$
$\overrightarrow{AB} = 2b$

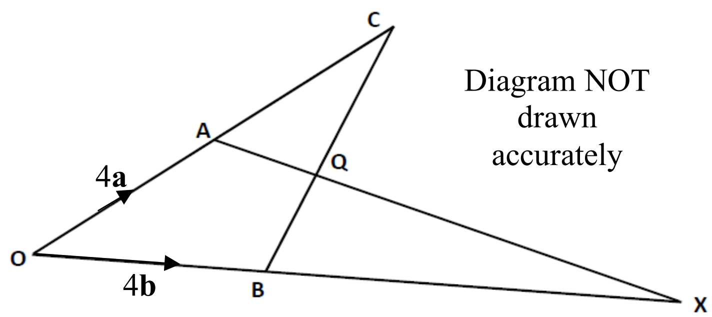
a) Calculate $\overrightarrow{MN}$ giving your answer in its simplest form.

(4)

b) Are the lines MN and AB parallel? Show all of your working.

(1)

Question 2



In the diagram

$\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$
A is the midpoint of OC
BQ:QC = 1:2
Find, in terms of **a** and **b**, the vector that represents

(a) $\overrightarrow{BC}$

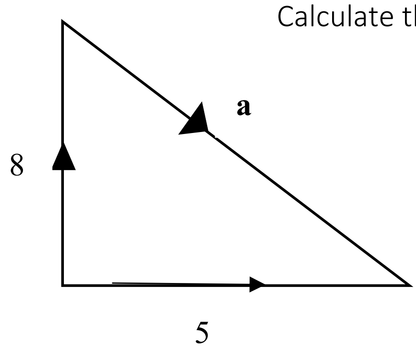
(1)

(b) $\overrightarrow{AQ}$

(2)

Question 3

Calculate the magnitude of vector **a.**



(2)

Total / 10

14 **Probability**

Question 1

Laura has 9 tins of soup in her cupboard, but all the labels are missing.

She knows that there are 5 tins of tomato soup and 4 tins of vegetable soup.

She opens three tins at random.

Work out the probability that she opens more tins of vegetable soup than tomato soup.

....

(4)

Question 2

A summer camp runs coasteering and surfing classes.

50 children attend the camp
35 children do coasteering
10 children do both classes
2 children do neither class

a) Draw a venn diagram to represent this information

$\mathscr{E}$

(2)

A child attending the summer camp is selected at random.

b) Find the probability that the child

i) did exactly one class

.... (2)

ii) did surfing, given that they did not do coasteering

.... (2)

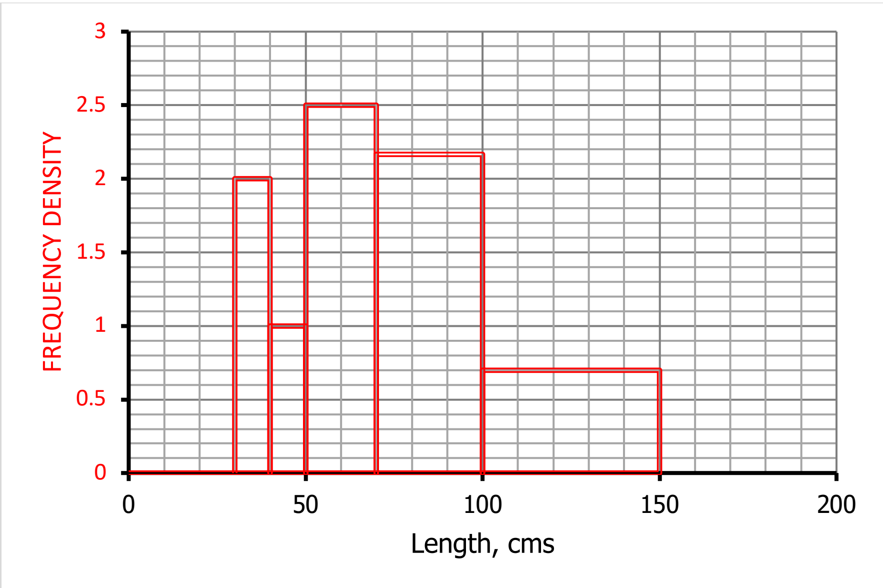
Total / 10

15 **Statistics**

Question 1

The table and histogram show the lengths of some pythons.

Length, cms			Frequency
30	< x ≤	40	20
40	< x ≤	50	10
50	< x ≤	70	50
70	< x ≤	100	
100	< x ≤	150	
		Total	



(a) Use the histogram to find the missing frequencies in the table

....
(2)

(b) Estimate the median python length.

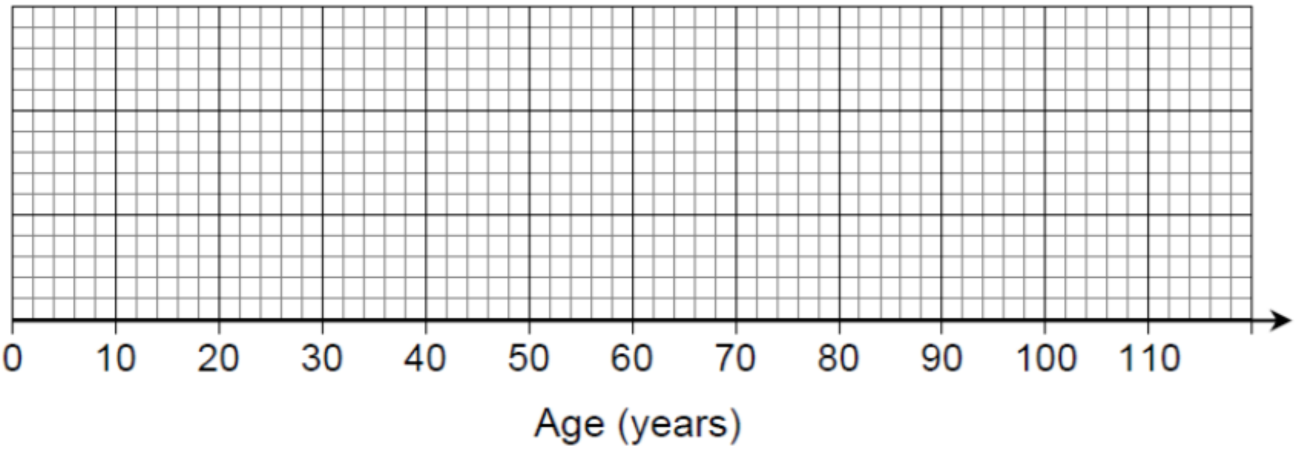
....
(3)

(Total 5 marks)

Question 2

In France in 2007 25% of the population were under 21years old. 50% were under 36. The interquartile range of the ages was 30 years. The oldest person was 103 years old.

a) Show this information on a boxplot



[3]

b) It is predicted that by 2040 the age distribution in France will have a lower quartile of 26 years, a median of 44 years and an upper quartile of 66 years.

Make two comments about the predicted change in the age distribution from 2007 to 2040.

Comment 1

Comment 2

(2)

(Total 5 marks)

Total / 10

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The Bridge to A level

Problem Solving

1 <u>Solving quadratic equations</u>

Question 1

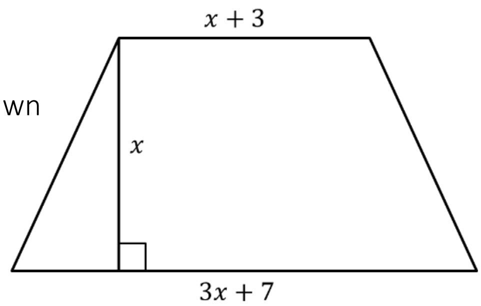
A number and its reciprocal add up to $\frac{26}{5}$.
Form and solve an equation to calculate the number.

....
(3)

Question 2

The diagram shows a trapezium.

Diagram **NOT** accurately drawn



All the measurements are in centimetres.
The area of the trapezium is 16 cm^2.

a) Show that $2x^2 + 5x - 16 = 0$

(1)

b) Work out the value of x to 1 decimal place.

$x =$.......................... (2)

Question 3

Two numbers have a product of 44 and a mean of 7.5.
Use an algebraic method to find the numbers.
You must show all of your working.

....
(4)

Total / 10

2 Changing the subject

Question 1

The surface gravity of a planet is given by $g = \frac{GM}{r^2}$ where

M = Mass of the planet
r = radius of the planet
G = gravitational constant $= 6.67x10^{-11}$

The surface gravity of Earth is $9.807 m/s^2$ and the mass of Earth is 5.98×10^{24}kg.

Find the radius of Earth in kilometres correct to 3 significant figures.

(4)

Question 2

In a parallel circuit, the total resistance is given by the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Make R_1 the subject of the formula

(4)

Question 3

Show that $\frac{1}{\frac{1}{x}+1} = \frac{x}{x+1}$

(2)

Total / 10

3 Simultaneous equations

Question 1

Sarah intended to spend £6.00 on prizes for her class but each prize cost her 10p more than expected, so she had to buy 5 fewer prizes.
Calculate the cost of each prize.

………………………
(5)

Question 2

Arthur and Florence are going to the theatre.
Arthur buys 6 adult tickets and 2 child tickets and pays £39.
Florence buys 5 adult tickets and 3 child tickets and pays £36.50.
Work out the costs of both adult and child tickets.

………………………
(5)

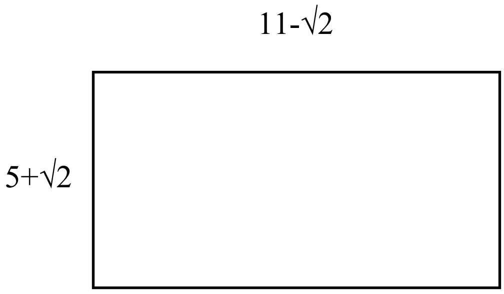
Total / 10

4 Surds

Question 1

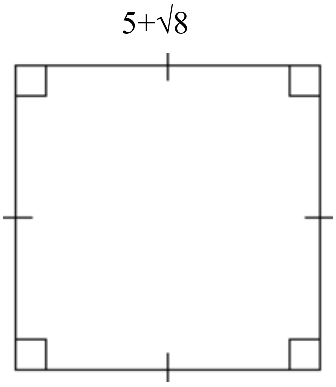
Calculate the area of each shape giving your answers in the form $a + b\sqrt{2}$

a)



(2)

b)



(3)

Question 2

Colin has made several mistakes in his 'simplifying surds' homework. Explain his error and give the correct answer.

i) $4\sqrt{3} \times 5\sqrt{12} = 20\sqrt{36}$

(2)

Question 3

The area of a triangle is $20cm^3$. The length of the base is $\sqrt{8}$cm. Work out the perpendicular height giving your answer as a surd in its simplest form.

(3)

Total / 10

5 **Indices**

Question 1

Lowenna says that $27^{-1/3} \times 64^{2/3}$ = 48

Is Lowenna correct? You must show all of your working.

(4)

Question 2

Which one of these indices is the odd one out? Circle your answer and give reasons for your choice.

$16^{-\frac{1}{4}}$ $\quad$ $64^{-\frac{1}{2}}$ $\quad$ $8^{-\frac{1}{3}}$

(2)

Question 3

Find values for a and b that make this equation work

$a^{\frac{1}{2}} = b^{\frac{1}{3}}$

(1)

Question 4

i) Write 25 as a power of 125

(1)

ii) Write 4 as a power of 32

(1)

iii) Write 81 as a power of 27

(1)

Total / 10

6 Properties of Lines

Question 1

(a) Write down the gradient of the line $2y - 4x = 5$.

(1)

(b) Write down the equation of a line parallel to $3y = 7 - 4x$.

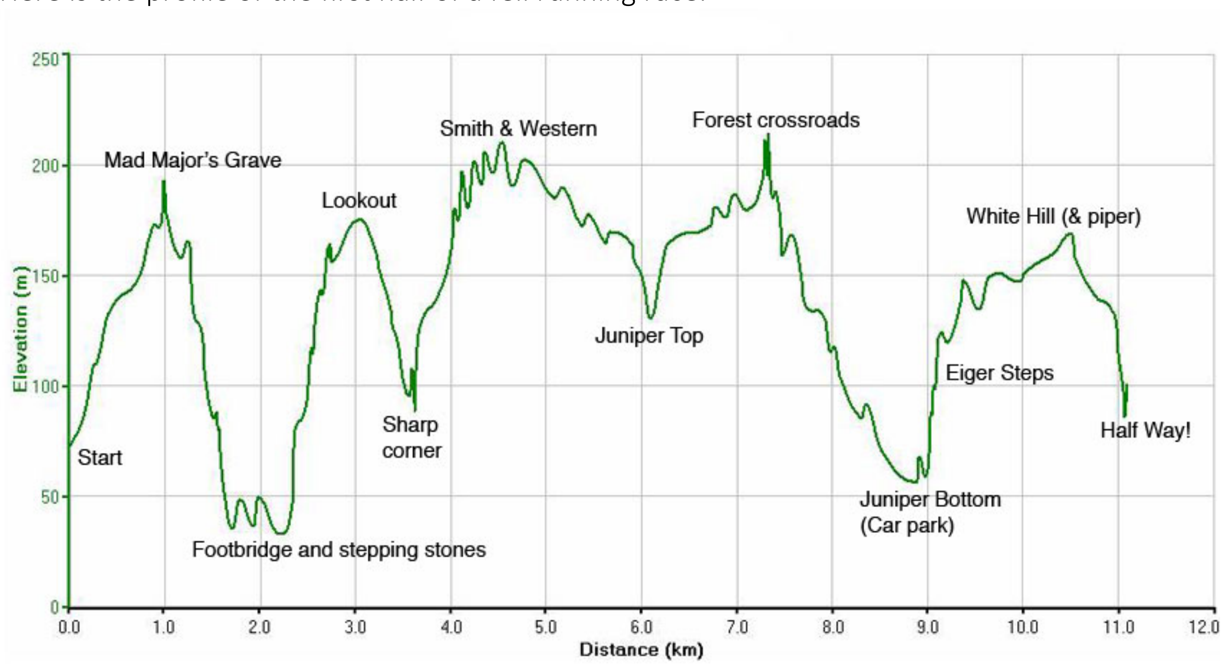
(1)

(c) Write down the equation of a line with gradient ½ and y-intercept of 6.

(1)

Question 2

Here is the profile of the first half of a fell running race.



(a) Work out the approximate gradient of the race from the start to Mad Major's Grave

(2)

(b) The most dangerous part of the race is from Mad Major's Grave to the Footbridge. Why do you think this might be?

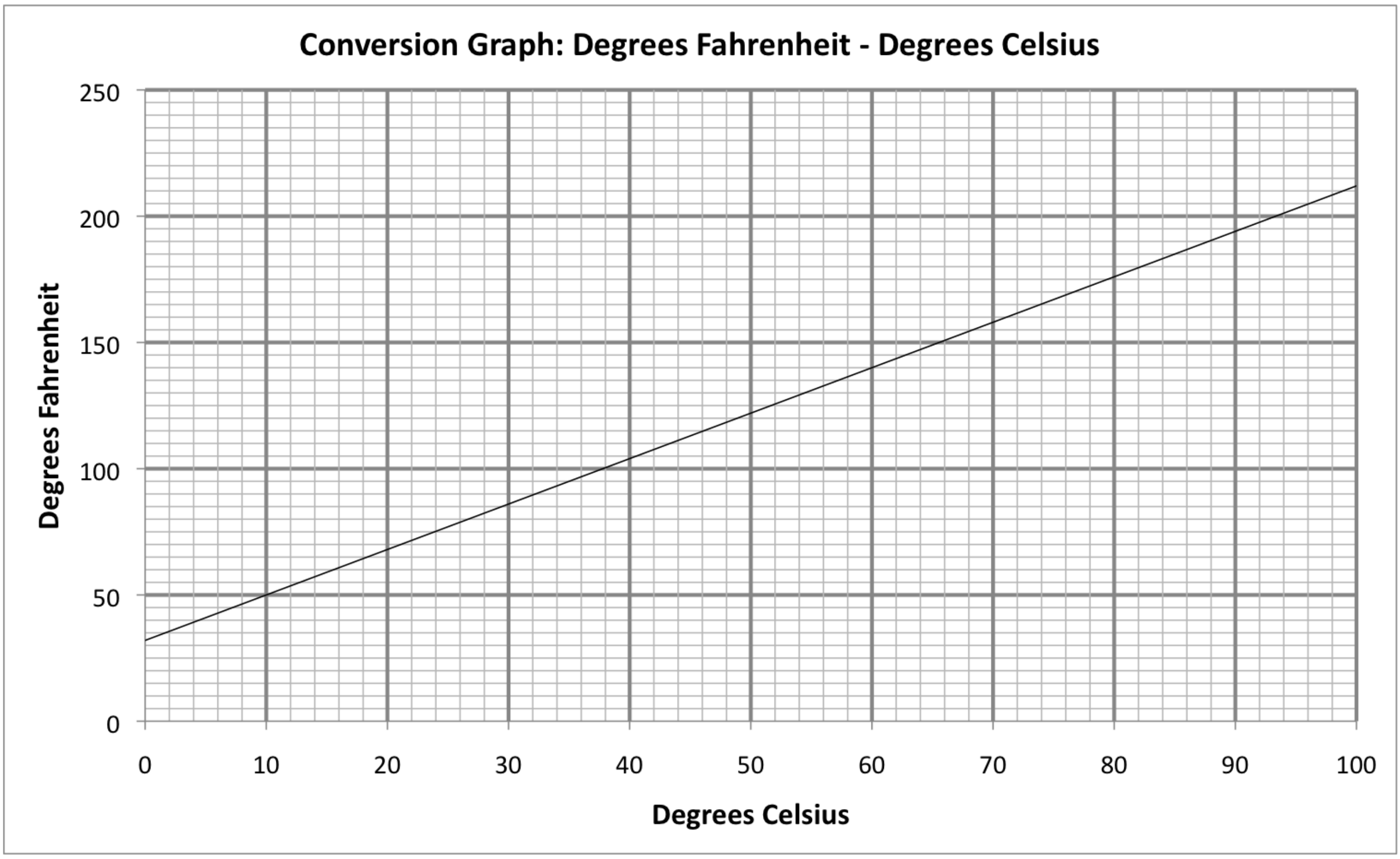
(1)

(c) Work out an estimate for the average ascent for the first four uphill sections of the race.

(2)

Question 3

Here is a graph used to convert degrees Celsius (C) and degrees Fahrenheit (F).



The equation of the straight line is given by $F = mC + a$
Calculate the values of m and a

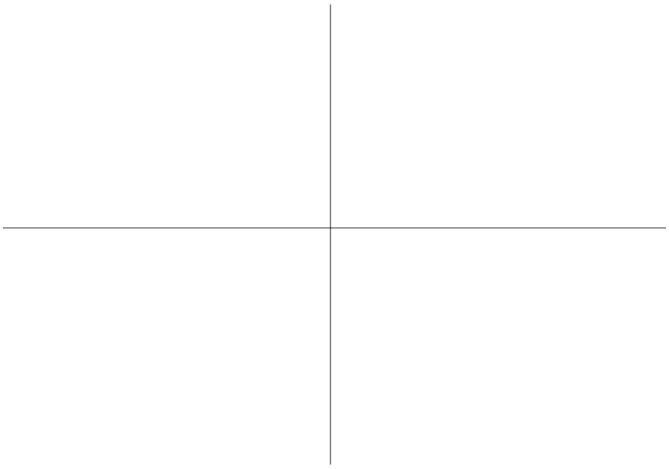
(2)

Total / 10

7 Sketching curves

Question 1

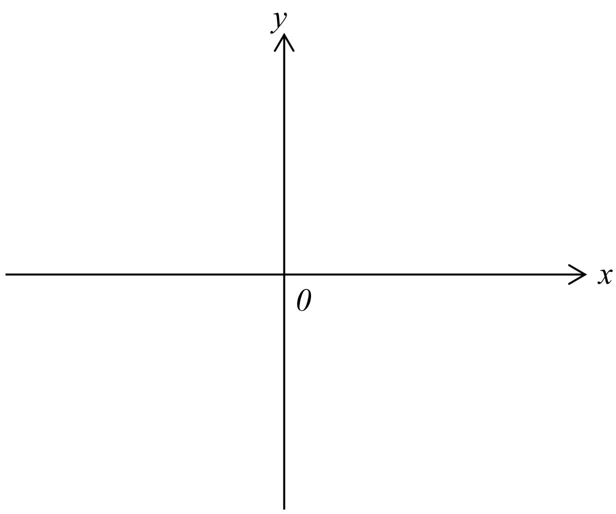
Sketch the graph of $f(x) = x^2 + 5x - 6$, showing the co-ordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(5)

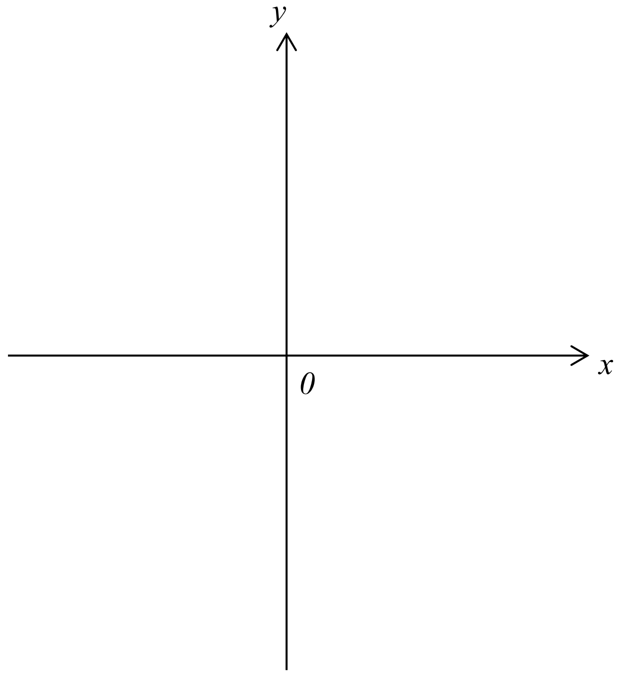
Question 2

a) On the axes sketch the graph of $y = \frac{3}{x}$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(2)

b) On the axes sketch the graph of $y = x^3 - 5$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(3)

Total / 10

8 Transformation of functions

Question 1

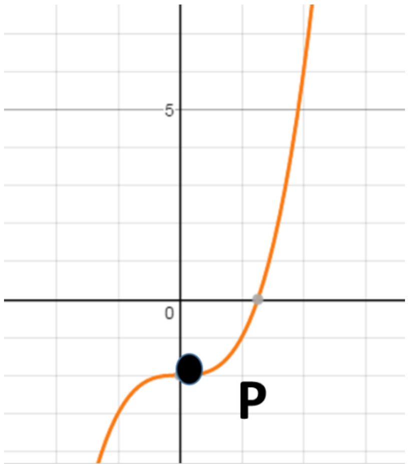
Here is a sketch of f(x).
The coordinates of P are (0,-2)
Sketch the graphs after the following translations and reflections, and state the coordinates of P':

a) g(x) = f(x) + 1

b) h(x) = f(x - 2)

c) j(x) = -f(x)

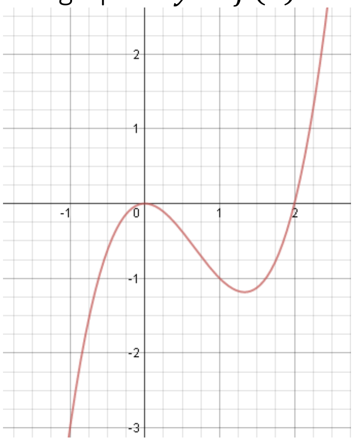
d) k(x) = f(-x)



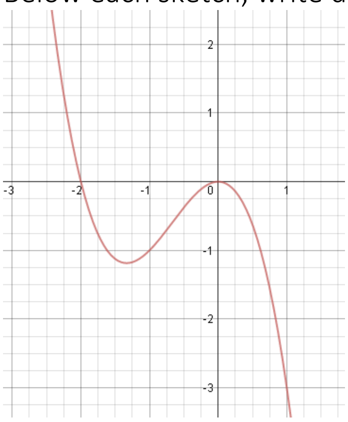
(4)

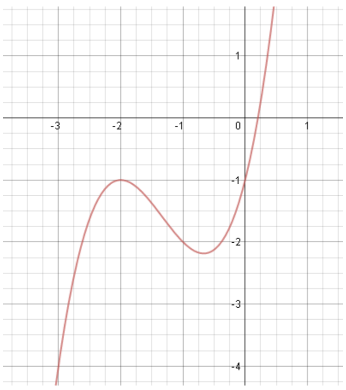
Question 2

The graph of $y = f(x)$ is shown below.



Below each sketch, write down the equation of the transformed graph





y =...................... *y* =........................

(2)

Question 3

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 4x + 5$
C is the minimum point of the curve.
(a) Find the coordinates of C after the transformation $f(x + 1) + 2$.

(………, ………)
(2)

(b) Hence, or otherwise, determine if $f(x - 3) - 1 = 0$ has any real roots.
Give reasons for your answer.

(2)

Total / 10

9 ## Pythagoras' theorem and Trigonometric ratios

Question 1

ABCDEFGH is a cuboid
AE = 5cm
AB = 6cm
BC = 9cm

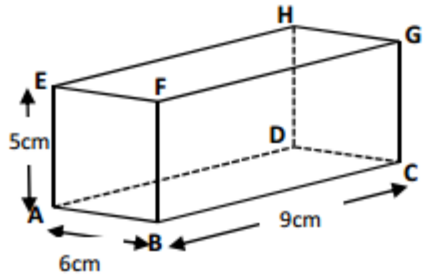


Diagram NOT drawn accurately

(a) Calculate the length of AG. Give your answer correct to 3 significant figures.

....

(1)

(b) Calculate the size of the angle between AG and the face ABCD.
Give your answer correct to 1 decimal place.

(3)

Question 2

A piece of land is the shape of an isosceles triangle with sides 7.5m, 7.5m and 11m.
Turf can be bought for £11.99 per 5m^2 roll.
How much will it cost to turf the piece of land?

....

(3)

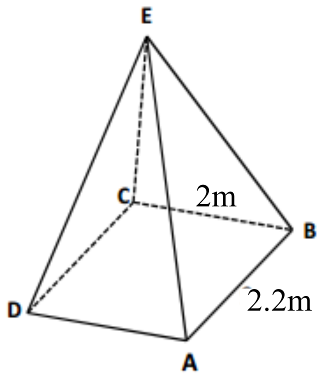
Question 3

Ben is 1.62m tall.

The tent he is considering buying is a square based pyramid.

The length of the base is 3.2m.

The poles AE, CE, AE and BE are 2m long.



Ben wants to know if he will be able to stand up in the middle of the tent. Explain your answer clearly.

....

(3)

Total / 10

10 Sine / Cosine Rule

Question 1

Plane A is flying directly toward the airport which is 20 miles away. The pilot notice a second plane, B, 45^o to her right. Plane B is also flying directly towards the airport. The pilot of plane B calculates that plane A is 50^o to his left. Based on that information how far is plane B from the airport? Give your answer to 3 significant figures.

……………………………

(4)

Question 2

Two ships, A and B, leave the same port at the same time.
Ship A travels at 35km/h on a bearing of 130^o.
Ship B travels at 25km/h on a bearing of 120^o.
Calculate how far apart the ships are after 1 hour.
Give your answer correct to two decimal places.

……………………………

(3)

Question 3

A farmer has a triangular field. He knows one side measures 450m and another 320m. The angle between these two sides measures 80^o. The farmer wishes to use a fertiliser that costs £3.95 per container which covers 1500m^2. How much will it cost to use the fertiliser on this field?

(3)

Total / 10

11 Inequalities

Question 1

A new cylindrical tube of snacks is being designed so that its height is 3 times its radius and its volume must be less than 20 times its radius.
Create an inequality and find possible values for the radius.

....
(3)

Question 2

A base jumper is going to jump off a cliff that is 50m tall, the distance she travels downwards is given by the equation

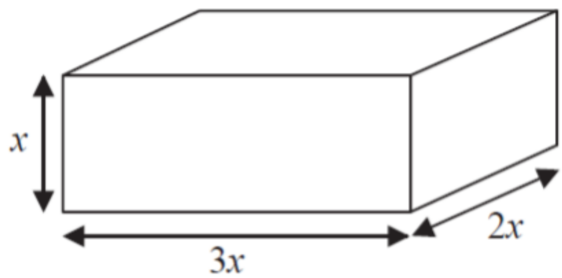
$$d = 4.9t^2 \quad \text{where} \quad t = \text{time of flight}$$
$$\text{and} \quad d = \text{distance travelled}$$

A video camera is set-up to film her between 20m and 10m above the ground.
Calculate the time period after the jumper jumps that filming taking place.

....
(3)

Question 3

The total volume of the box is less than 1 litre.
Given that all lengths are in cm and that x is an integer,
Show that the longest side is less than 18cm.



(4)

Total / 10

12 Algebraic proof

Question 1

Katie chooses a two-digit number, reverses the digits, and subtracts the smaller number from the larger.

For example

42 – 24 = 18

She tries several different numbers and finds the answer is never a prime number.

Prove that Katie can never get an answer that is a prime number.

(4)

Question 2

Here are the first 5 terms of an arithmetic sequence

1 6 11 16 21

Prove that the difference between the squares of any 2 terms is always a multiple of 5.

(6)

Total / 10

13 Vectors

Question 1

OAB is a triangle

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

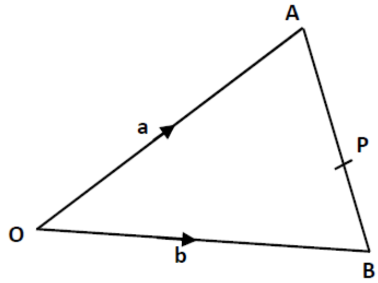


Diagram NOT drawn accurately

....

(1)

P is the point on $\overrightarrow{AB}$ such that AP: PB = 3:2

(b) Show that $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

....

(3)

Question 2

OABC is a parallelogram.

X is the midpoint of OB

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

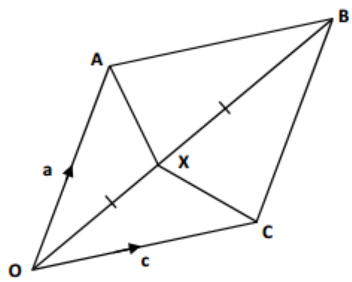


Diagram NOT drawn accurately

(a) Find the vector $\overrightarrow{OX}$ in terms of **a** and **c.**

....

(1)

(b) Find the vector $\overrightarrow{XC}$ in terms of **a** and **c.**

....

(2)

Question 3

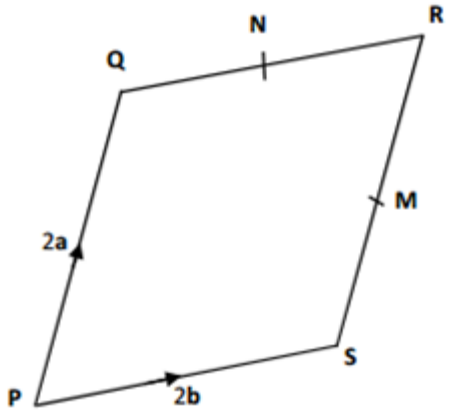


Diagram NOT drawn accurately

PQRS is a parallelogram.
M is the midpoint of RS
N is the midpoint of QR

$\overrightarrow{PQ} = 2\mathbf{a}$

$\overrightarrow{PS} = 2\mathbf{b}$

Use vectors to proof that the line segments SQ and MN are parallel.

(3)

Total / 10

14 Probability

Question 1

Max has an empty box.

He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1 : 3.

Julie takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{19}{316}$.

How many red counters did Max put in the box?

(5)

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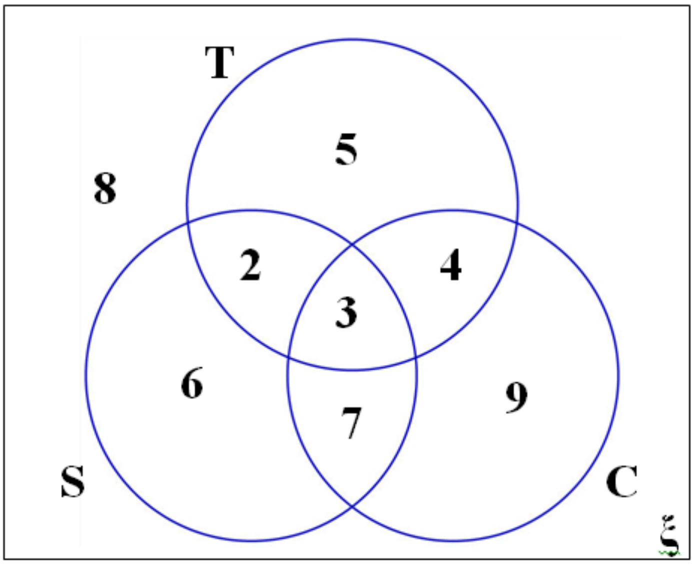
Question 2

The Venn diagram shows the ice-cream flavours chosen by a group of 44 children at a party.

The choices are strawberry (S), choc-chip (C) and toffee (T).

A child is picked at random.



Work out :

(a) P(S)

....

(1)

(*b*) P(T ∪ C | C)

....

(2)

(*c*) P(C | S ∪ T)

....

(2)

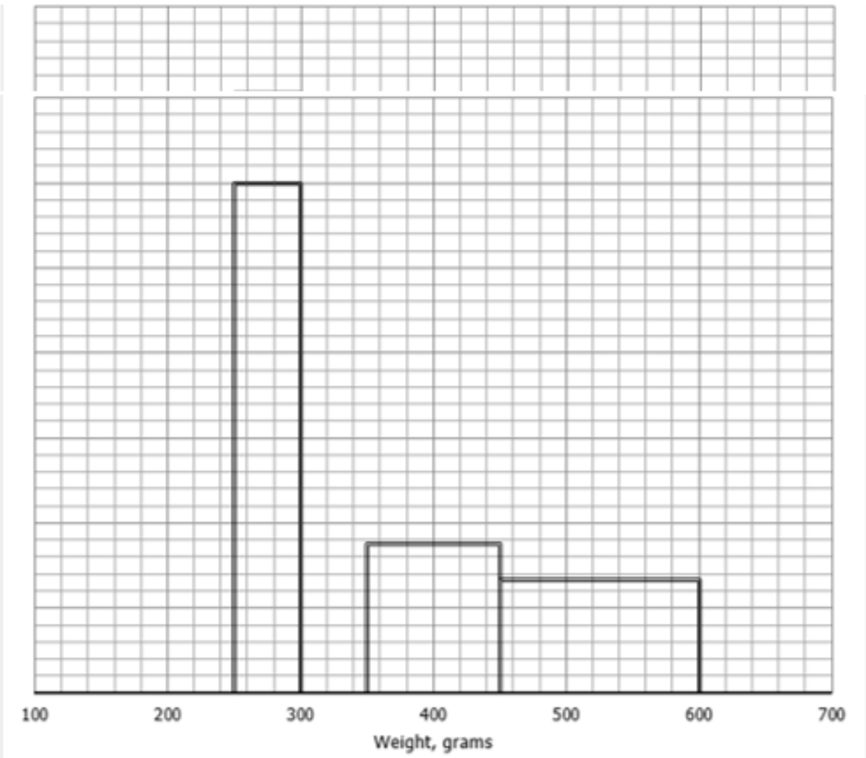
Total / 10

15 <u>Statistics</u>

Question 1

The table and histogram show the weights of some snakes.

Weight, grams	Frequency
250 < x ≤ 300	60
300 < x ≤ 325	25
325 < x ≤ 350	40
350 < x ≤ 450	35
450 < x ≤ 600	40
Total	200



(a) Use the information to complete the histogram

(3)

(b) Calculate an estimate for the median

....

(2)

Question 2

Sarah played 15 games of netball. Here are the number of goals she scored in each game.

17 17 17 18 19 20 21 22 24 25 25 26 28 28 28

a) Draw a boxplot to show this information

(3)

a) Lucy plays in the same 15 games of netball. The median number of points Lucy scores is 24. The interquartile range of these points is 10 and the range of these points is 17.

Who is the better player, Sarah or Lucy?
You must give a reason for your answer.

(2)

Total / 10